

Question:

A mass of 3 kg – initially at rest on a table - experiences a time dependant force $F(t)=(1+t)^2$, where F is in Newtons and t is in seconds.

- How far has the object traveled when $t=3$?
- What is the change in the object's velocity from $t=8$ to $t=10$?
- What acceleration does the box experience at $t=5$ seconds?

Numerical Answers:

- 6.75 m
 - 63.54m/s
 - 12m/s^2
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Reasoning:

Force = Mass(acceleration) – find acceleration, the rest is kinematics.

$$F(t)=(1+t)^2 \therefore a(t) = \frac{F(t)}{m} = \frac{(1+t)^2}{m}$$

$$v = \int a(t)dt = \int \frac{(1+t)^2}{m} dt$$

Solve by u-substitution or by direct integration of the polynomial

Direct method:

$$\frac{1}{m} \int 1 + 2t + t^2 (dt) = \frac{1}{m} [t + t^2 + \frac{1}{3}t^3]$$

and the integration of the velocity function gives the position

$$\frac{1}{m} \int t + t^2 + \frac{1}{3}t^3 (dt) = \frac{1}{m} [\frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{12}]$$

So, when $t = 3$ the displacement is

$$\begin{aligned} \frac{1}{m} \int t + t^2 + \frac{1}{3}t^3 (dt) &= \frac{1N}{3kg} [\frac{3^2}{2} + \frac{3^3}{3} + \frac{3^4}{12}] \\ &= \frac{1N}{3kg} [4.5 + 9 + 6.75] = 6.75m \end{aligned}$$

The change in velocity from $t=8$ to $t=10$ can be written as

$$\begin{aligned} \frac{1}{3} \{ [10 + 10^2 + \frac{1}{3}10^3] - [8 + 8^2 + \frac{1}{3}8^3] \} \\ \frac{1}{3} [433.3 - 242.6] = 63.54m / s \end{aligned}$$

which shows an increase consistent with an accelerated object.

$$a(5) = (1+5)^2/3 = 12\text{m/s}^2$$