Question:

A particular unstable isotope is at rest and decays radioactively into a proton and two alpha particles. The alpha particles have a mass four times as large as the proton. Of the total energy released in the event, some is distributed amongst the particles as kinetic energy. The proton acquires most of this kinetic energy. If the remainder of the kinetic energy is divided equally between the alpha particles, derive an expression for the magnitude and direction of the alpha particle velocities with respect to the origin of the event. *Hint: Exploit as much symmetry as possible and use symbols as much as possible. Explicitly define new ones as needed to simplify the solution. Assume the event is 2-dimensional.*

Numerical Answer:

Reasoning:

Ok. Energy is NOT conserved, but clearly only internal forces are acting during the decay process. Thus, momentum is conserved. This is basically a restatement of the 'ball explodes into three parts' example in the text.

There is no initial momentum in either **I** or **j** directions. Since the kinetic energy 'left over' is divided equally among the alpha particles, they must both have the same speed, and therefore make the same angle with respect to the origin of the explosion,

And so,

$$\vec{p}_o = 0, \vec{p}_f = 0$$

$$p_{fx} = m_p v_p(+\hat{i}) + 2m_\alpha v_{\alpha x}(-\hat{i})$$

$$p_{fy} = m_\alpha v_{\alpha y}(-\hat{j}) + m_\alpha v_{\alpha y}(+\hat{j})$$

Using the **i** equations yields

$$m_p v_p = 2m_\alpha v_\alpha \cos\theta$$

The **j** direction yields no new information

Using WET yields the following relationship

$$\tfrac{1}{2}(E_{released} - K_{proton}) = K_\alpha = \tfrac{1}{2}m_\alpha v_\alpha^2$$

so

$$\frac{(E_{released} - K_{proton})}{m_\alpha} = v_\alpha^2$$

and

$$\left(\frac{m_p v_p}{2m_\alpha \cos\theta}\right)^2 = (v_\alpha)^2$$

so

$$\left(\frac{m_p v_p}{2m_\alpha \cos\theta}\right)^2 = \frac{(E_{released} - K_{proton})}{m_\alpha}$$

$$\left(\frac{m_p v_p}{2m_\alpha \cos\theta}\right)^2 = \frac{(E_{released} - \frac{1}{2}m_p v_p^2)}{m_\alpha}$$

With a little rearranging this becomes

$$\cos\theta = \sqrt{\left(\frac{K_p}{2m_\alpha(E - K_p)}\right)}$$

where $K_p = \frac{1}{2}m_p v_p^2$

So, finally…each alpha particle has a speed given by

$$\sqrt{\frac{(E_o - K_p)}{m_\alpha}} = v_\alpha \text{ at } \theta = \cos^{-1}\sqrt{\left(\frac{K_p}{2m_\alpha(E - K_p)}\right)}$$

with respect to the line of symmetry defined by the direction of proton emission