

Question:

1. A neutron ($m_n = 1.675 \cdot 10^{-27}$ kg) is accelerated to a final speed of $6 \cdot 10^4$ m/s and then strikes a stationary beryllium atom ($M_{Be} = 14.97 \cdot 10^{-27}$ kg). If the collision can be considered elastic and one-dimensional, find the direction and magnitude of the post-collision velocities of the neutron and beryllium. Solve in the Center of Mass frame of reference.

Numerical Answer: The neutron has a final velocity of $4.79 \cdot 10^4$ m/s in the (-x) direction

Reasoning:

Using our conventional reference frame:

Elastic implies that kinetic energy is conserved as well as momentum being conserved. Thus

$$\frac{1}{2} m_n v_{nO}^2 = \frac{1}{2} m_n v_{nf}^2 + \frac{1}{2} m_{Be} v_{Bef}^2$$

$$m_n v_{nO}^2 = m_n v_{nf}^2 + m_{Be} v_{Bef}^2$$

The velocity of the center of mass

$$V_{CM} = \frac{m_n v_{nO} + m_{Be} v_{BeO}}{m_n + m_{Be}}$$

$$V_{CM} = \frac{m_n v_{nO}}{m_n + m_{Be}} (+\hat{x})$$

In the center of mass frame prior to the collision

$$v'_n = v_{nO} - V_{CM}$$

$$v'_{Be} = v_{BeO} - V_{CM}$$

After the collision in the COM frame

$$v'_n = -(v_{nO} - V_{CM})$$

$$v'_{Be} = -(v_{BeO} - V_{CM})$$

Finally we transform back to the original frame of reference

$$v_{nf} = v'_n + V_{CM} = -(v_{nO} - V_{CM}) + V_{CM}$$

$$v_{Bef} = v'_{Be} + V_{CM} = -(v_{BeO} - V_{CM}) + V_{CM}$$

So we now have

$$v_{nf} = 2 \frac{m_n v_{nO}}{m_n + m_{Be}} - v_{nO} = v_{nO} \left(\frac{2m_n}{m_n + m_{Be}} - 1 \right)$$

$$v_{Bef} = 2 \frac{m_n v_{nO}}{m_n + m_{Be}} - v_{BeO} = v_{nO} \left(\frac{2m_n}{m_n + m_{Be}} - \frac{v_{BeO}}{v_{nO}} \right)$$

when we substitute numbers

$$v_{nf} = 6 \cdot 10^4 \left(\frac{(2)(1.675)}{1.675 + 14.97} - 1 \right) (m/s)$$

$$v_{nf} = 4.79 \cdot 10^4 (m/s) (-\hat{x})$$

and

$$v_{Bef} = 6 \cdot 10^4 \left(\frac{(2)(1.675)}{1.675 + 14.97} - \frac{0}{6 \cdot 10^4} \right) (m/s)$$

$$v_{Bef} = 1.207 \cdot 10^4 (m/s) (+\hat{x})$$