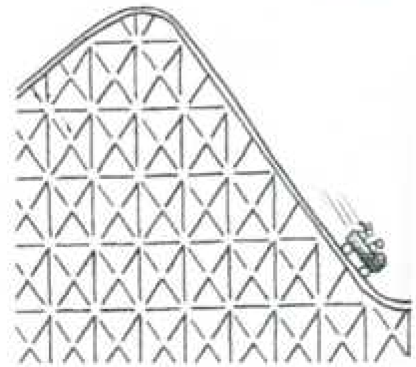
Physics 195 **Fall 2009** **Problem Set 6**

Show all calculations. Explain all assumptions. Answer in standard MKS units.
Explicitly substitute units into your symbolic equations to verify solution.
Express answers in 5 or fewer digits. Use scientific notation as appropriate.

Conceptual Questions: **Place the letter** corresponding to your answer **in the box.**
Limit your explanation to the space provided. Please write legibly.

1. A roller coaster is pulled to the top of a steep incline and allowed to roll down. To increase the thrill factor, you want to increase the speed of the car at the bottom of the hill by a factor of two. In order to do so you must increase the height of the hill by a factor of



$\boxed{C}$

a) two
b) three
c) four
d) six
e) eight
Explain

since $mgh = \frac{1}{2}mv^2$
then to have $v \to 2v$
$v^2 \to 4v^2$
$\therefore\ mg(4h) = \frac{1}{2}m(2v)^2$

$4h$

10

2. On a foggy day, completely identical cars A (moving at ~70 mph) and B (moving at ~50 mph) are side by side when they see an overturned truck on the road ahead. Both immediately slam on the brakes in an effort to avoid crashing into the truck. Car B skids to a halt less than an inch from the truck. From this information, the speed of car A when it hits the truck is closest to

$\boxed{D}$

a) 20 mph
b) 30 mph
c) 35 mph
d) 50 mph
e) Cannot be answered without more information.
Explain

$V_{fB}^2 = \emptyset \quad V_{0B}^2 - 2a\Delta x = \emptyset \ \therefore\ V_{0B}^2 = 2a\Delta x$

$V_{fA}^2 = V_{0A}^2 - 2a\Delta x \rightarrow$ with $\cancel{V_{0A}}^2 = \cancel{V_{0B}}^2$ then...

$V_{fA}^2 = V_{0A}^2 - V_{0B}^2 \Rightarrow V_{fA}^2 \approx 70^2 - 50^2 \simeq 50^2$

10

Show all calculations. Explain all assumptions. Answer in standard MKS units.
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3. Tracks A and B are made from pieces of iron that have the same length. They are completely identical. Then, a small dip is made in the middle of track B. Two small, identical balls are simultaneously released at the top of both tracks. The ball that reaches the end of the track first is on...

B

a) Track A
b) Track B
c) Each reaches the end of their respective track at the same time
d) Depends on other information

Explain:

while crossing the dip, ball B has an average speed > ball A ∴ ball B finishes 1st

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4. A brick is lifted to a given height and dropped to the ground. Next, a second brick (which weighs twice as much as the first) is lifted to the same height and dropped to the ground. When the second brick lands, it has...

C

a) Half as much kinetic energy as the first
b) The same kinetic energy as the first
c) Twice as much kinetic energy as the first
d) Four times as much kinetic energy as the first

Explain:

since $K = \frac{1}{2}mv^2$ and

$v = \sqrt{2gH}$ then since v same

doubling m doubles K

10

since... $F = -\frac{du}{dx}$ where ~~when~~ the minus indicates the direction of the force ... (decreasing potential)

$U(x) = \frac{x^4}{5} - 15x^3 + 125x^2 + 100$

$\frac{du}{dx} = \frac{4}{5}x^3 - 45x^2 + 250x$. where this is zero indicates points of equilibrium

$\emptyset = \frac{4}{5}x^3 - 45x^2 + 250x \Rightarrow \emptyset = \frac{4}{5}x^2 - 45x + 250$

$x = \frac{45 \pm \sqrt{45^2 - 4(\frac{4}{5})(250)}}{8/5} \Rightarrow \frac{5}{8}\left(45 \pm \sqrt{45^2 - 800}\right)$

$x = \emptyset$, $x = \frac{5}{8}(45 \pm 35) \Rightarrow \frac{5}{8}(80)$ and or $\frac{5}{8}(10)$

$x = \emptyset$, $x = 50$, $x = 6.25$ $\qquad x = 50$ lies outside domain

check $\qquad \boxed{x = \emptyset, \ x = 6.25 \text{ cm}}$

equilibrium stability → second derivative test

$\frac{d^2u}{dx^2} = \frac{12}{5}x^2 - 90x + 250$ $\qquad$ @ $x = \emptyset$ $\frac{d^2u}{dx^2} > \emptyset$

@ $x = 6.25$ $\frac{d^2u}{dx^2} < \emptyset$

$\therefore$ only stable equilibrium position @ $\boxed{x = 0}$

can only be 'trapped' if the kinetic energy @ $x = 6.25$ is zero, since $v = \emptyset$ represents a turning point.

$$U(6.25) = U_f = \frac{x^4}{5} - 15x^3 + 125x^2 + 100 \text{ (eV)}$$

$$U(\emptyset) = U_0 = 100 \text{ (eV)}$$

$$U_f - U_0 = K_0 - K_f \qquad K_f \to \emptyset \text{ b/c } v = \emptyset \text{ @ 'turning' point}$$

$$\Delta U = \frac{x^4}{5} - 15x^3 + 125x^2 = \frac{1}{2}mv^2$$

$$v_{max} = \sqrt{\frac{2}{9.11 \cdot 10^{-31}\,\text{kg}}\left(\frac{6.25^4}{5} - 15(6.25)^3 + 125(6.25)^2\right)}$$

(parenthesis = $\cancel{4928.85}$ 1525.9 eV $\times 1.6 \times 10^{-19} \frac{J}{eV} = 7.89 \cdot 10^{-16}$

$$\boxed{v_{max} = 2.32 \cdot 10^7 \text{ m/s}}$$

if @ top of bump $F_N = \frac{mg}{c}$ then the forces must satisfy $\sum F(\hat{r}) = \frac{mv^2}{R}(-\hat{r})$. So...

$\sum F(\hat{r}) = F_N(+\hat{r}) + mg(-\hat{r}) = \frac{mv_1^2}{R_1}(-\hat{r})$ so then with $F_N =$

$$\frac{mg}{c} - mg = -\frac{mv_1^2}{R_1} \Rightarrow mg\left(1-\frac{1}{c}\right) = \frac{mv_1^2}{R_1}$$

$R_1 g\left(1-\frac{1}{c}\right) = v_1^2 \Rightarrow$ speed @ top of bump.

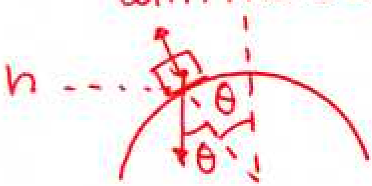
using COE $\Rightarrow Lmg\sin\Theta_H = mgR_1 + \frac{1}{2}mv_1^2$

$$Lmg\sin\Theta_H = mgR_1 + \frac{1}{2}mgR_1\left(1-\frac{1}{c}\right)$$

$$L\sin\Theta_H = R_1 + \frac{1}{2}R_1\left(1-\frac{1}{c}\right) \Rightarrow R_1\left(1+\left(\frac{c-1}{2c}\right)\right)$$

$$L\sin\Theta_H = R_1\left(\frac{3c-1}{2c}\right) \Rightarrow \boxed{R_1 = \left(\frac{2c}{3c-1}\right)L\sin\Theta_H}$$

Rider becomes projectile when it loses contact with the surface.



$\sum F(\hat{r}) \Rightarrow F_N(+\hat{r}) + mg\cos\Theta(-\hat{r}) = \frac{mv_2^2}{R_2}(-\hat{r})$

$\therefore R_2 g\cos\Theta_L = v_2^2$ <critical speed>

using COE $\Rightarrow Lmg\sin\Theta_H = \frac{1}{2}mv_2^2 + mgh$

with $v_2^2 = R_2 g\cos\Theta_L$; $h = R_2\cos\Theta_L$ then

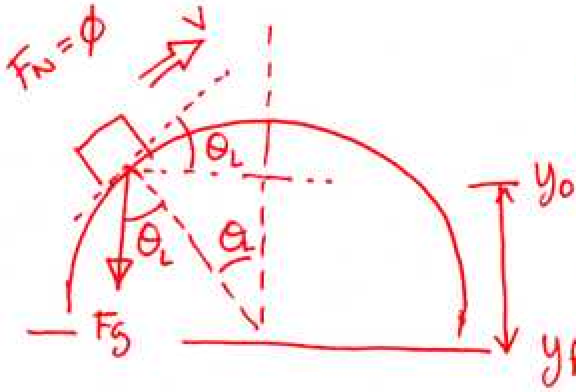
$$Lmg\sin\Theta_H = \frac{1}{2}mR_2 g\cos\Theta_L + mgR_2\cos\Theta_L$$

$$L\sin\Theta_H = \frac{R_2}{2}\cos\Theta_L + R_2\cos\Theta_L$$

$$L\sin\Theta_H = \frac{3}{2}R_2\cos\Theta_L \quad \text{with } R_2 = \frac{R_1}{b}$$

$$\frac{2b}{3R_1}L\sin\Theta_H = \cos\Theta_L \Rightarrow \frac{1}{R_1} = \frac{3c-1}{(2c)(L\sin\Theta_H)}$$

$(3c-1)(b)$



$y_0 = R_2 \cos\theta_L (+\hat{y}) \qquad V_{0x} = V_2 \cos\theta_L$

$V_{0y} = V_2 \sin\theta_L (+\hat{y})$

$a_y = g(-\hat{y})$

$$y_f - y_0 = V_{0y} t - \frac{g}{2} t^2 \qquad t = \frac{V_{0y} \pm \sqrt{V_{0y}^2 + 2 g y_0}}{g}$$

since $\sqrt{\ } > V_0$, must take (+) root for $t > \phi$

$$t = \frac{V_2 \sin\theta \pm \sqrt{V_2^2 \sin^2\theta + 2 g y_0}}{g} \qquad V_2^2 = R_2 g \cos\theta$$

so $$t = \frac{(R_2 g \cos\theta)^{\frac{1}{2}} \sin\theta + \sqrt{R_2 g \cos\theta \sin^2\theta + 2 g R_2 \cos\theta}}{g}$$

but since $\sin^2 = 1 - \cos^2$ then...

$$t = \frac{(R_2 g \cos\theta)^{\frac{1}{2}} \sin\theta + \sqrt{R_2 g \cos\theta (1 - \cos^2\theta) + 2 R_2 g \cos\theta}}{g}$$

lets work within the square root....

$$R_2 g \cos\theta - R_2 g \cos\theta \cos^2\theta + 2 R_2 g \cos\theta =$$

$$3 R_2 g \cos\theta - R_2 g \cos\theta \cos^2\theta = R_2 g \cos\theta\, (3 - \cos^2\theta)$$

$$t = \frac{(R_2 g \cos\theta)^{\frac{1}{2}} \sin\theta + \sqrt{R_2 g \cos\theta (3 - \cos^2\theta)}}{g}$$

$$= \frac{(R_2 g \cos\theta)^{\frac{1}{2}} \sin\theta + (R_2 g \cos\theta)^{\frac{1}{2}} (3 - \cos^2\theta)^{\frac{1}{2}}}{g}$$