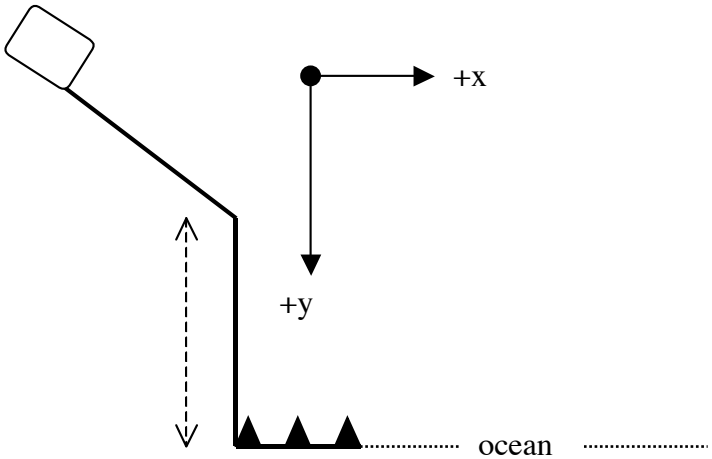
Question:

A penguin is at rest at the top of a long smooth slope inclined at a 15 degree angle. The penguin has a mass of 2.75 kg. At the end of the slope is a 20 m vertical cliff. At the base of the cliff is a section of rocky ground 10 m long. Beyond the rocks lies the ocean. What is the minimum distance from the edge of the cliff that the penguin should start to slide in order to miss the rocks and make it into the ocean? (use coordinate system provided though you may rotate it as needed)



Numerical Answer:

a) About 605 cm

Reasoning:

This is a disguised projectile motion problem. We need to find the launch speed that will allow the penguin to clear the rocks, then determine what distance will provide this final speed. So…

$$\Delta y = 20m(+y) = v\sin 15 t(+y) + \tfrac{1}{2} g t^2 (+y)$$

$$\Delta x = 10m(+x) = v\cos 15 t(+x)$$

$$\therefore \frac{10m}{v\cos 15} = t = \frac{10.4}{v}$$

$$\Delta y = 20m(+y) = v\sin 15 \frac{10.4}{v}(+y) + \tfrac{1}{2} g (\frac{10.4}{v})^2 (+y)$$

$$20m(+y) = v(0.259)(\frac{10.4}{v})(+y) + (\tfrac{1}{2} g)(\frac{10.4}{v})^2 (+y)$$

$$20 = (2.69) + (\tfrac{1}{2} g)(\frac{10.4}{v})^2$$

$$20 = 2.69 + \frac{531}{v^2}$$

$$v = 5.54 m/s$$

So, how does the penguin achieve this speed? By sliding down the hill. We know the final speed, we know the initial speed, we know the acceleration so we can find the distance.

$$v_f^2 = v_0^2 + 2a\Delta L$$

$$v_f = 5.54 m/s, v_o = 0$$

$$30.68 = 2 \times g\sin 15 \times \Delta L$$

$$\Delta L = \frac{30.68}{5.07}$$

$$\Delta L = 6.05m$$