

One of the more important points to emerge from this discussion is the understanding that the shape of the potential energy function reveals a lot of information about the magnitude and direction of the forces that act upon the system.

Of particular interest are ‘Central Potentials’. This term describes potential energy functions that are symmetric about a point and change as a function of distance from that point. Central potentials are created by forces that are symmetrically applied about that same point. These sorts of forces are found in the study of both gravitation and electromagnetism.

The attractive force that exists between objects possessing mass is called the gravitational force. It has been experimentally determined to be proportional to the total mass in a system, and inversely proportional to the distance separating the masses. The force produced is of the form

$$F_G = \frac{Gm_1m_2}{r_{12}^2}(-\hat{r})$$

where G is Newton’s Universal Gravitational Constant. The direction of the force is always directed inwards towards the center of the reference mass in the radial direction, as indicated by the negative sign.

We observe that gravity acts as a conservative force and use the work-energy theorem to find the gravitational potential energy function:

$$W = \int \frac{Gm_1m_2}{r_{12}^2} dr = \frac{-Gm_1m_2}{r_{12}}$$

As work is equal to the change the energy of a system, this shows that an object making a transition from one radial distance to another at a constant velocity undergoes a change in potential energy given by:

$$\Delta U = -Gm_1m_2 \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

As with any system, in order to define changes, we need a reference point. We choose our reference point of zero energy at infinity, reasoning that any two object separated by an infinite distance will not be able to interact, and thus the energy of the system is a constant. With the zero set at infinity, then the potential energy function decreases as one object moves towards the other. In other words, attractive systems have negative potential energy, and tend to move towards regions of lower potential energy, reaching a minimum when the two objects are in contact.

If the system has a total energy which is negative then the system is called a bound system and the objects are trapped within, unable to escape. A total system energy of zero means that the object has escaped from the system, but has zero speed when it does so. Finally, positive system energy means the object has escaped with some non-zero speed.

The relationship between the force and the potential energy function can be summarized as follows:

$$F(r) = \frac{-dU}{dr}$$

This states that forces act in the direction of decreasing potential energy. Thus, objects tend to ‘fall down the potential hill’ trading a decrease in potential energy for an increase in kinetic energy.

The similarities between the gravitational interactions of masses and the electrical interactions of charges are numerous. You will encounter central potentials again in the study of electrostatics, and again in the development of the quantum atomic model. In each case, the essential behavior of the system remains the same.