

In the beginning, there was an observation that some objects are moving and others are not. If an object is moving, it covers a particular distance in some time and ends up being displaced from its initial position. Immediately we must define both distance and displacement. Although we use the two words interchangeably in our daily language, in the language of physics they are two completely different quantities.

Recall that vectors have both a magnitude and direction, while scalars have only a magnitude. “25 miles per hour” is a scalar quantity, but saying “25 miles per hour heading southwest” indicates we are dealing with a vector. Distance is a scalar quantity indicating the total path length, but displacement is the spatial difference between the initial and final locations of the object and is thus a vector.

It is important to note that since vectors are used to describe measurable and physically meaningful quantities, there is no such thing as a ‘negative’ vector. A negative number is a number ‘less than’ zero. Since vectors are representative of measurable quantities, all vectors have magnitudes greater than zero but they may lie in a direction that has been arbitrarily labeled as negative in a particular coordinate system.

A coordinate system provides a reference for the various quantities. The choice of a coordinate system will have a tremendous impact on the way in which problem solutions are stated, and some problems are greatly simplified by the proper choice of coordinate systems. An ill-defined coordinate system will return meaningless results.

The concept of vectors will form the cornerstone of the study of physics, and one of the first clues we need to obtain is whether the quantity of interest behaves as a vector or a scalar. If you gloss over this first step, it is unlikely that you will be able to complete the course with a satisfactory grasp of the essential concepts.

We apply our knowledge of mathematics to understand and define the relationship between displacement, velocity and acceleration. With average velocity defined as the displacement of the object over a certain time period, then the use of calculus allows for mathematical descriptions of instantaneous velocity:

$$\vec{v}_x = \frac{d\vec{x}}{dt}$$

and instantaneous acceleration

$$\vec{a}_x = \frac{d\vec{v}_x}{dt} = \frac{d^2\vec{x}}{dt^2}$$

For the special case of constant acceleration we are able to derive the three main equations of linear motion:

$$\Delta\vec{x} = x_F - x_O = \vec{v}_{OX}t + \frac{1}{2}\vec{a}_x t^2$$

$$v_{FX}^2 = v_{OX}^2 + 2\vec{a}\Delta\vec{x}$$

$$\vec{v}_{FX} = \vec{v}_{OX} + \vec{a}_x t$$

These vector equations form the basis of our investigations, allowing us to determine the future position, velocity and acceleration of some object.

Once the ideas of one-dimensional motion are absorbed, we realize that we can incorporate additional directions of motion so long as we carefully add subscripts to our equations in order to differentiate between the various coordinate axes.

Adding vector quantities in different dimensions requires that we develop means by which to add, subtract, and multiply these vectors.

The basic way of adding vectors is to find the net components of the vectors in the various directions and then utilize the Pythagorean Theorem to obtain a value for the magnitude of the resultant vector:

$$|r| = \sqrt{\Delta x^2 + \Delta y^2} ,$$

but this is not enough to specify a vector quantity, for we need some direction with respect to a reference line:

$$\tan \theta_r = \frac{\Delta y}{\Delta x}$$

There are two other vector operations that will prove useful. The operation used to multiply two vectors and obtain a scalar is called the dot product:

$$\vec{A} \bullet \vec{B} = |A| |B| \cos \theta_{AB}$$

The other vector multiplication operation produces a vector perpendicular to both the original vectors and is called the cross product:

$$\vec{A} \times \vec{B} = |A| |B| \sin \theta_{AB}$$

The direction of the resultant vector may be found by using the right hand rule we develop in our analysis of rotating systems, or by evaluating the determinant of a simple matrix.