

With the understanding of linear motion, we turn our attention to the origin of the acceleration in our systems. Following Newton, we define acceleration as the result of a force applied to a mass. A force is initially described as a ‘push’ or a ‘pull’ applied by one object making contact with another, but we move past this point to develop the concept of a field.

A field is used to explain how one object is able to exert a force on another object without contact. This is the ‘action at a distance’ effect described in various texts. We explore the earth’s gravitational field as a means to understand this concept. We come to realize that gravitational force is the interaction of a mass and an external gravitational field. This important concept will be used again in our investigation of electricity and magnetism. All forces may be explained as the interaction of a particular property of the object with the appropriate field, a topic explored further in quantum field theory.

Now we begin with an analysis of the various forces in a system. Applying the ideas of Newton allows us to find the acceleration of a system. Once the acceleration of the system is known, all the quantities derived in our exploration of motion are solvable.

To properly analyze the various forces requires an appropriate coordinate system and the use of Newton’s Laws. This set of mathematical relationships gives us a starting point from which to begin our investigations. The net force acting on a mass produces a net acceleration:

$$\sum \vec{F}_{system} = \sum m_{system} \vec{a}_{system}$$

When the net acceleration of the system is zero, then the system is said to be in equilibrium with respect to its inertial reference frame. There are two types of equilibrium. In static equilibrium, the vector sum of the forces acting on the object equal zero and the object has no velocity. In dynamic equilibrium, the forces

still sum to zero, but the object is moving with a constant velocity.

The one exception to our rule regarding dynamic equilibrium is the special case of uniform circular motion. Since moving along a circular path implies that the direction of the velocity vector must continually change, this means that there must be some unbalanced acceleration acting on the object. This acceleration is called the ‘centripetal’ acceleration because the acceleration must act perpendicular to the velocity vector and towards the center of the circular path in order for circular motion to be possible.

In our analysis of all simple rotating systems we must identify the net force that is directed towards the center of the circle. No matter what provides that net force, it is equivalent to the magnitude of the centripetal force.

In any analysis, it is very important to understand and differentiate between a force and a net force. While an object may well be at rest in a reference frame, that does not imply that there are no forces that act upon it. Rather, the most we can say about the situation is that there is no net force acting upon that object. In other words, the object behaves as though it were not experiencing a force, but that does not imply that there are no forces acting upon it.

Until this point, we have assumed that the various forces acting on a system produce linear motion. We refine this to suggest that forces acting along the center of mass of a system produce linear motion.

To locate the center of mass in a body of uniform density, locate the geometric center. For objects of non-uniform density, the use of calculus produces the center of mass equation:

$$R_{CM} = \frac{1}{M_{SYSTEM}} \int_0^R r dm$$

where r is defined as the distance from the origin of the coordinate system to the point

An overview of classical mechanics

where a mass element dm is located. How dm depends on the geometry and density functions of the object is listed here:

one-dimensional: $dm = \lambda ds$

λ = linear mass density (kg/m) function

ds = section of the 1-D integration path

two-dimensional: $dm = \sigma dA$

σ = surface mass density (kg/m²) function

dA = section of the 2-D integration path

three-dimensional: $dm = \rho dV$

ρ = mass density (kg/m³) function

dV = section of the 3-D integration path

When we find the net force acting on the center of mass of a system, we are able to use the equations of motion developed earlier. This means we are able to calculate the behavior of an entire system of objects, so long as we are able to describe the center of mass.