

There is a property of moving masses even more fundamental than kinetic energy. This vector property, called momentum, is the product of mass and velocity. For systems in which mass is constant it becomes clearer when expressed as follows:

$$\frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}$$

This relation shows that momentum changes of an object are due to the force acting on that object. This is another way of stating Newton's Law of Inertia. The most general statement involves the possibility of changing both mass and velocity.

$$\frac{d\vec{p}}{dt} = m \frac{dv}{dt} + v \frac{dm}{dt}$$

The triumph of momentum conservation is that it enables the study of systems *even when the exact forces acting within the system are unknown*. So long as there are no external forces acting on the system, then momentum is conserved even when energy is not! This is a fantastic result and will serve as the basis for most of our understanding of the interactions of objects.

We label the interaction of two objects as a collision and classify the collision by whether or not energy is conserved. Elastic collisions are interactions where energy is conserved during the process. Perfectly inelastic collisions are ones in which the objects are 'stuck' together and move as single object following the collision. Between elastic and completely inelastic collisions lie the 'semi-elastic' collisions.

To characterize the type of collision, we make use of the coefficient of restitution, which is a measure of the ratio of the relative velocities of the objects immediately before and after the collision.

$$\epsilon = \frac{-(v_{2f} - v_{1f})}{(v_{2o} - v_{1o})}$$

It is at this point that the discussion of the concept of 'Center of Mass' returns to help simplify the study of collisions. If we examine elastic collisions in this reference frame, we find that the post collision velocities are simply the opposite of the pre-collision velocities! This result greatly simplifies the resulting calculations. Once again, the frame of reference of the problem affects the solution, a point returned to again and again.