Forces directed along axes that do not pass through the center of mass of a free object do not cause pure translational motion, but a mixture of rotation and translation. So, to understand these more complicated motions, we first analyze pure rotational motion and then by using the principles of energy and momentum conservation we are able to examine mixed systems.

Just as in linear spaces, we need to define the basic relationships between displacement, velocity and acceleration. We imagine a rigid rod that is constrained to move in a horizontal circle. While the path length swept out by a point on the rod will vary depending on the distance from the axis of rotation, each point on the rigid body sweeps out the same angle in the same time.

Thus, we define the unit of angular displacement to be radians. The angle swept out in one time increment defines the angular velocity, and the rate of change of angular velocity is the angular acceleration. The development of rotational dynamics will follow the same steps as those used in linear dynamics.

With angular displacement defined as $\Delta\theta$ then angular velocity becomes:

$$\vec{\omega} = \frac{d\vec{\theta}}{dt}$$

and angular acceleration is thus:

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

Each time an object of uniform radius completes a full rotation, a point on the rim of the object has moved through a path length of $2\pi r$. Therefore we can see that angular and linear displacements are related by the radius of the rotating object.

$$d\vec{s} = r d\vec{\theta}$$

If displacements are related, then so are velocity and acceleration:

$$\vec{v} = r\vec{\omega}$$

$$\vec{a} = r\vec{\alpha}$$

The equations of motion for rotating systems must obey the same dimensional constraints as the equations of motion for translating systems.

$$\Delta\vec{s} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$

$$r\Delta\vec{\theta} = r\vec{\omega}_o t + \tfrac{1}{2}r\vec{\alpha}t^2$$

$$\therefore \Delta\vec{\theta} = \vec{\omega}_o t + \tfrac{1}{2}\vec{\alpha}t^2$$

Similarly, the other one-dimensional equations of motion may be derived:

$$\omega_f^2 = \omega_o^2 + 2\vec{\alpha}\Delta\vec{\theta}$$

$$\vec{\omega}_f = \vec{\omega}_o + \vec{\alpha}t$$

Given the similarities, it appears that the kinetic energy of rotation is related to the kinetic energy of translation.

$$K_T = \tfrac{1}{2}mv^2$$

$$K_R = \tfrac{1}{2}m(\omega^2 r^2)$$

rearranging the terms slightly leads to:

$$K_R = \tfrac{1}{2}(mr^2)\omega^2 = \tfrac{1}{2}I\omega^2$$

In this expression I is called the moment of rotational inertia – moment of inertia. It plays the same role as mass in the formulation of Newton's Laws as it resists angular acceleration. Mass appears as a property of matter that resists being put into linear motion, and the moment of inertia appears as a property of matter that resists it being put into rotational motion.

For objects capable of being modeled by a point mass, the term (mr^2) represents the moment of inertia. For objects larger than point masses, the exact value of the moment of inertia depends on the total mass of the object, as well as how that mass is arranged around the axis of rotation. In general, the moment of inertia of any object can be written as:

$$I = \int r dm$$

Once again, the relationships of density and geometry will affect the method by which to express the mass element *dm*.