

Once the concept of inertial mass has been adjusted to account for rotations, the comparison between the translational and rotational frames of reference leads to the concepts of angular momentum and rotational forces. In particular,

$$\vec{L} = I\vec{\omega}$$

$$\vec{\tau} = I\vec{\alpha}$$

With torque as the rotational counterpart to force, a torque applied to an object with a particular moment of inertia will result in a particular angular acceleration. Aside from the change of units and variables, the analysis of rotating systems proceeds just as it did for linear systems. We are still able to predict the displacement, velocity and acceleration of an object in the rotating coordinate frame.

Under certain conditions, the displacement of the object in the rotational frame of reference can be related to the displacement in a translational reference frame. The special condition is called rolling without slipping, and refers to a system where the center of mass of the rolling object displaces a distance of  $2\pi R$  with each rotation. In such systems, the translational velocity and the angular velocity are related by the radius of the rolling object.

Under these conditions, the displacement, velocity and acceleration in the rotational reference frame may be directly related to these same quantities in the translational frame of reference:

$$\Delta\vec{x} = R\Delta\vec{\theta}$$

$$\vec{v}_x = R\vec{\omega}$$

$$\vec{a}_x = R\vec{\alpha}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

To revisit the topics of static and dynamic equilibrium, we now must add the conditions that for an object to be in static equilibrium both the sum of the forces and the sum of the

torques acting on the object must be zero. This ensures a static object with no translational or rotational motion. The dynamic equilibrium condition now is modified to include constant rotational speeds in addition to constant translational speeds.

This also means that when dealing with a system, we must account for both rotational and translational components of energy, momentum, velocity, acceleration, displacement, and so on before we are able to make accurate predictions as to the future state of the system. It is this very notion that eventually leads to the development of the Bohr atomic model and then the quantum model of the atom.