

finish off §1.4 plus preview of §2.2 Rate of Change

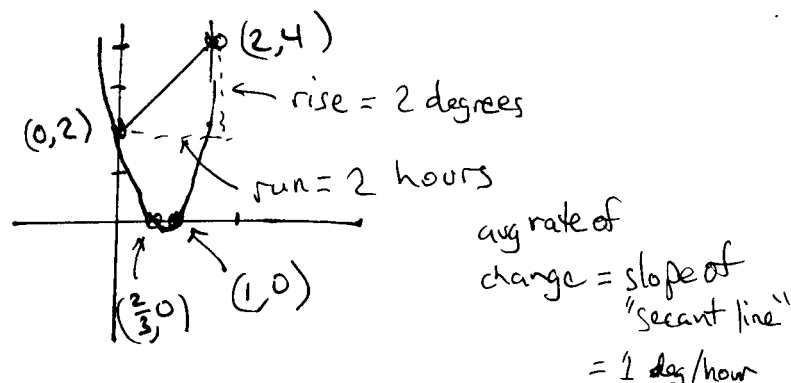
ex [from 1.4 #66] $f(x) = 3x^2 - 5x + 2 = (x-1)(3x-2)$

	x	y
	-1	10
	0	2
	1	0
change in time = 2 - 0 = 2 hours	2	4
	3	14

change in temp
= 4 - 2
= 2 degrees

zeros: 1 , $\frac{2}{3}$

x-intercepts: $(1, 0)$ $(\frac{2}{3}, 0)$



How much does

$f(x)$ change between $x=0$ and $x=2$?

At hour zero, $f(0) = 2$ degrees C.

At hour 2, $f(2) = 3 \cdot 2^2 - 5 \cdot 2 + 2 = 12 - 10 + 2 = 4$ deg C.

Net change = $4 - 2$ degrees = 2 degrees

What is the average rate of change of the temp over that time span? (deg/hr)

$$\frac{f(2) - f(0)}{2 - 0} = \frac{2 \text{ degrees}}{2 \text{ hours}} = 1 \text{ degree/hour}$$

ex (cont'd): What is the change in temp
between hour zero and hour h ?

$$\begin{aligned}\text{change in temp} &= f(h) - f(0) \\ &= [3h^2 - 5h + 2] - 2 \\ &= 3h^2 - 5h\end{aligned}$$

In particular, when $h=2$, this change is
 $3 \cdot 2^2 - 5 \cdot 2 = 12 - 10 = 2$ degrees, as before.

What is the average rate of change (degrees per hour)
between $x=0$ and $x=h$?

$$\begin{aligned}\frac{f(0+h) - f(0)}{(0+h) - 0} &= \frac{f(h) - f(0)}{h - 0} = \frac{3h^2 - 5h}{h} \\ &= \frac{h(3h - 5)}{h} = 3h - 5\end{aligned}$$

In particular if $h=2$ hours,

the avg rate of change is $3(2) - 5 = 1$ deg/hour

But if $h=1$ hour

the avg rate of change is $3(1) - 5 = -2$ deg/hour

ex (cont'd) For $f(x) = 3x^2 - 5x + 2$

a) what is the change in temp between
time = x and time = $x+h$

b) what is the avg rate of change in temp.
between these two times?

answer to a) $f(x+h) - f(x) = \{\text{temp at } x+h \text{ hours}\} - \{\text{temp at } x \text{ hours}\}$

(Step 1)

$$\begin{aligned} f(x+h) &= 3(x+h)^2 - 5(x+h) + 2 \\ &= 3(x^2 + 2hx + h^2) - 5x - 5h + 2 \\ &= 3x^2 + 6hx + 3h^2 - 5x - 5h + 2 \end{aligned}$$

(Step 2)

$$\begin{aligned} f(x+h) - f(x) &= (3x^2 + 6hx + 3h^2 - 5x - 5h + 2) \\ &\quad - (3x^2 \qquad \qquad \qquad - 5x \qquad \qquad + 2) \\ &= 3x^2 + 6hx + 3h^2 - 5x - 5h + 2 \\ &\quad - 3x^2 \qquad \qquad \qquad + 5x \qquad \qquad - 2 \\ &= 6hx + 3h^2 - 5h \quad \text{"difference quotient"} \\ &= h(6x + 3h - 5) \end{aligned}$$

answer to b)

$$\text{avg rate of change} = \frac{\text{change in temp}}{\text{change in time}} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

(Step 3)

$$= \frac{h(6x + 3h - 5)}{h} = \boxed{6x + 3h - 5}$$

In particular, when $x=0$, this is $3h-5$