2.1 Limits and Continuity

ex: $f(x) = \dfrac{x^2-4}{x-2}$ ← Domain = all real numbers except $x=2$

But we are interested in values of x when x is "near" 2.

x	$f(x)$
1.9	3.9
1.99	3.99
1.999	3.999
2	undefined
2.001	4.001
2.01	4.01
2.1	4.1

Scratch work

$$f(2.1) = \frac{(2.1)^2-4}{(2.1)-2} = \frac{4.41-4}{2.1-2} = \frac{0.41}{0.1} = 4.1$$

$$f(2.01) = \frac{(2.01)^2-4}{(2.01)-2} = \frac{4.0401-4}{2.01-2} = \frac{0.0401}{0.01} = 4.01$$

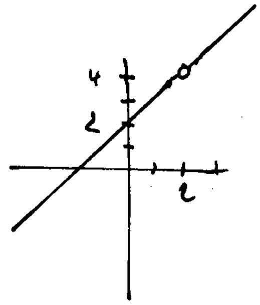
Remark: Hey wait a minute! $f(x) = \dfrac{x^2-4}{x-2} = \dfrac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \begin{cases} x+2 & \text{if } x \neq 2 \\ \text{undefined} & \text{if } x=2 \end{cases}$

So $f(2.001) = 2.001 + 2 = 4.001$

$f(1.999) = 1.999 + 2 = 3.999$

Graph of $f(x) = \dfrac{x^2-4}{x-2}$ Same as $y = x+2$

but there's a "hole" at $x=2$



We say $\lim\limits_{x \to 2} f(x) = 4$.

(Hand-waving)

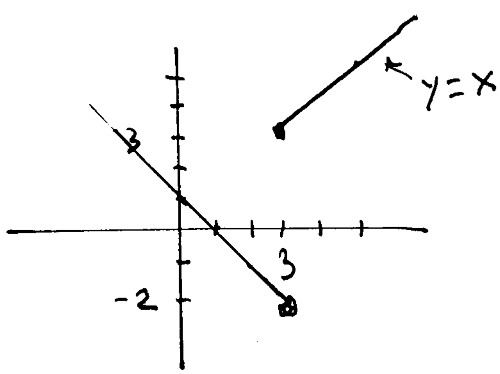
<u>Defn</u>: The statement $\lim\limits_{x \to c} f(x) = L$ means

that when x gets closer and closer to c,

y gets closer and closer to L.

<u>ex</u>: $f(x) = \begin{cases} x & \text{if } x \geq 3 \\ -x+1 & \text{if } x < 3 \end{cases}$

Does $\lim\limits_{x \to 3} f(x)$ exist? $\boxed{\text{No.}}$



x	$f(x)$
2.99	$-2.99+1 = -1.99$
2.999	-1.999
3	3
3.001	3.001
3.01	3.01
3.1	3.1

In this circumstance, we say (i) $\lim\limits_{x \to 3^+} f(x) = 3$ ("right-side limit")

and (2) $\lim\limits_{x \to 3^-} f(x) = -2$ ("left-side limit")

<u>Fact</u>: $\lim\limits_{x \to c} f(x)$ <u>exists</u> if and only if both <u>one</u>-sided limits exist <u>and</u> they are <u>equal</u>.

ex: [algebraic tricks to help find limits]

$$f(x) = \frac{x^3-27}{x-3}$$ Does $\lim_{x\to 3} f(x)$ exist?

If so, what is the limit?

Note: If we try to plug in $x=3$, we get $\frac{0}{0}$ which is undefined

Recall the factoring formulas $a^3-b^3 = (a-b)(a^2+ab+b^2)$
[and $a^3+b^3 = (a+b)(a^2-ab+b^2)$]

$$f(x) = \frac{x^3-3^3}{x-3} = \frac{(x-3)(x^2+3x+9)}{x-3} = \begin{cases} x^2+3x+9 & \text{if } x\neq 3 \\ \text{undef.} & \text{if } x=3 \end{cases}$$

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} x^2+3x+9 = 3^2+3(3)+9$$
$$= 9+9+9 = 27$$

Remark: Because of Laws of Limits, in most cases

$$\lim_{x\to c} f(x) = f(c).$$

ex: $$\lim_{x\to 0} \frac{x^3-27}{x-3} = \frac{0^3-27}{0-3} = \frac{-27}{-3} = 9$$

ex: $$\lim_{h\to 0} \frac{-1}{(2+h)\cdot 2} = \frac{-1}{(2+0)\cdot 2} = -\frac{1}{4}$$

2.1 #16) $\lim_{t \to 3} \sqrt[3]{t^2+t-4} = \sqrt[3]{3^2+3-4}$

$= \sqrt[3]{8} = 2$

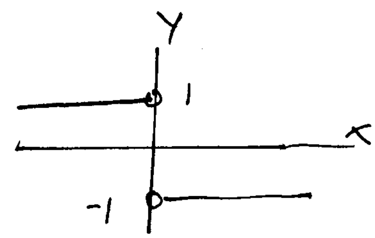
22) $\lim_{h \to 0} (2x^2+4xh+h^2)$

$= 2x^2+4x(0)+0^2 = 2x^2$

32) $\lim_{h \to 0} \frac{x^2h - xh^2 + h^3}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(x^2 - xh + h^2)}{\cancel{h}}$

$= x^2 - x(0) + 0^2 = x^2$



44) $f(x) = -\frac{|x|}{x}$

a) Find $\lim_{x \to 0^-} f(x) = 1$

b) " $\lim_{x \to 0^+} f(x) = -1$

c) " $\lim_{x \to 0} f(x)$ does not exist,

because $1 \neq -1$.

x	y
-2	1
-1	1
0	undef.
1	-1
2	-1