Loose end in § 2.1 Continuity

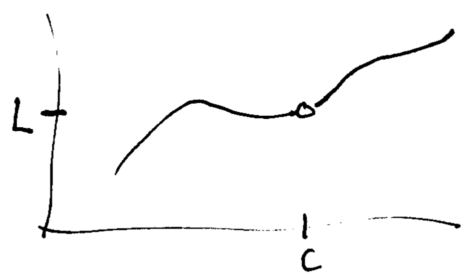
Definition. A function f is continuous at $x=c$ if

1. $f(c)$ is defined (that is, c is in the domain of f)

2. $\lim_{x \to c} f(x)$ exists

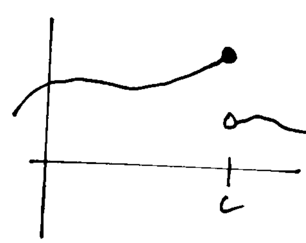
3. $\lim_{x \to c} f(x) = f(c)$.

graphical examples



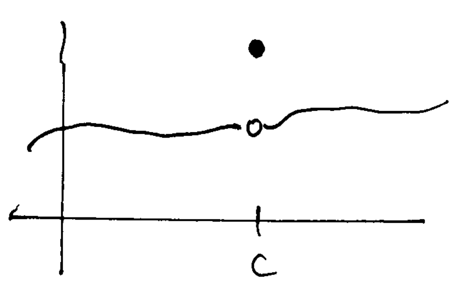
Not continuous

$f(c)$ is not defined



Not continuous

$\lim_{x \to c} f(x)$ does not exist



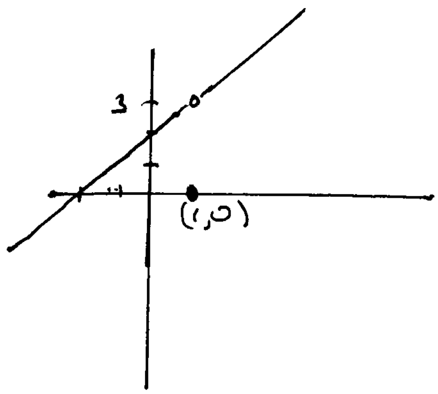
$f(c)$ ~~exists~~ is defined ✓

$\lim_{x \to c} f(x)$ exists ✓

But the two values are not equal.

ex: $f(x) = \begin{cases} x+2 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$

where (if anywhere) is this function discontinuous?



$f(1) = 0$

$\lim_{x \to 1} f(x) = 3$

and $0 \neq 3$.

Remark: If, say, f is continuous at every x value on the interval $(2,5)$, we say f say "f is continuous on the interval $(2,5)$".

2.2 Rates of Change, Slopes, and derivatives

example: x = time (hours)

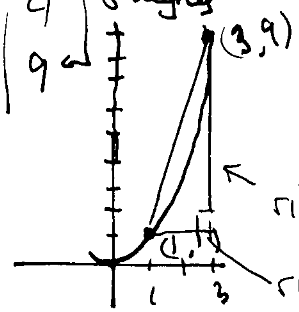
$f(x) = x^2$ = temperature (degrees)

What is the average rate of ~~the~~ change of temp. between $x = 1$ and 3 hours?

x	$f(x)$
1	1
2	4
3	9

2hrs

8 degrees



$$\frac{\text{change in temp}}{\text{change in time}} = \frac{f(3)-f(1)}{3-1} = \frac{3^2-1^2}{3-1} = \frac{8 \text{ degrees}}{2 \text{ hours}}$$

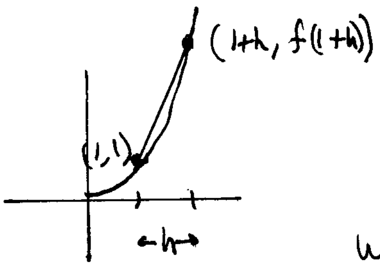
$= 4$ deg/hr

slope of the secant line

Now find the average rate of change between hour 1, and hour 1+h, so $x=1$ to $x=1+h$. Because $f(x)=x^2$,

$$\text{avg rate of change} = \frac{\text{change in temp}}{\text{change in time}} = \frac{f(1+h)-f(1)}{(1+h)-1}$$

$$= \frac{(1+h)^2 - 1^2}{(1+h)-1} = \frac{1+2h+h^2-1}{h} = \frac{2h+h^2}{h}$$



$$= \frac{\cancel{h}(2+h)}{\cancel{h}} = 2+h \ \text{degrees/hour}$$

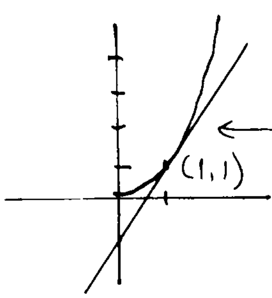
what happens when h gets closer and closer to 0 hours?

That is, $\lim\limits_{h\to 0} \dfrac{f(1+h)-f(1)}{h} = \ ?$

Answer: $\lim\limits_{h\to 0} \dfrac{f(1+h)-f(1)}{h} = \lim\limits_{h\to 0} 2+h$

$= 2$ degrees/hour

$=$ instantaneous rate of change of temp when $x = 1$ hour



Tangent line at (1,1) has slope = 2 degrees/hour