2.4 Product and Quotient Rules

Product Rule

Q: Is the derivative of product = product of derivatives?

exaple: $y = x^3 \cdot x^5 = x^8$

$$\frac{d}{dx}[x^8] = 8x^7$$

But $\frac{d}{dx}[x^3] \cdot \frac{d}{dx}[x^5] = (3x^2)\cdot(5x^4)$

$= 15x^6$

NOPE! In fact, not even the degree is correct.

Product Rule

$$\frac{d}{dx}[f \cdot g] = f' \cdot g + f \cdot g'$$

$$\boxed{\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)}$$

exaple:

$$\frac{d}{dx}[x^3 \cdot x^5] = \frac{d}{dx}[x^3] \cdot x^5 + x^3 \cdot \frac{d}{dx}[x^5]$$

$$= 3x^2 \cdot x^5 + x^3 \cdot 5x^4$$

$$= 3x^7 + 5x^7 = 8x^7$$

that is, the answer we were expecting

2.4 10) Find $f'(x)$ and simplify

$$f(x) = 6\sqrt[3]{x}(2x+1)$$
$$= 6x^{1/3}(2x+1)$$

$$f'(x) = \frac{d}{dx}[6x^{1/3}]\cdot(2x+1) + (6x^{1/3})\cdot\frac{d}{dx}[2x+1]$$
$$= 6\cdot\frac{1}{3}x^{-2/3}\cdot(2x+1) + 6x^{1/3}\cdot 2$$
$$= 2x^{-2/3}(2x+1) + 12x^{1/3}$$

optional $\rightarrow$ $$= \frac{2(2x+1)}{\sqrt[3]{x^2}} + 12\sqrt[3]{x}$$

18) $f(x) = (2x-1)(1-x^2)$

$$f'(x) = \frac{d}{dx}[2x-1]\cdot(1-x^2) + (2x-1)\cdot\frac{d}{dx}[1-x^2]$$
$$= 2\cdot(1-x^2) + (2x-1)\cdot(-2x)$$
$$= 2-2x^2-4x^2+2x$$
$$= -6x^2+2x+2$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

ex: $\frac{d}{dx}\left[\frac{x^8}{x^3}\right] = \frac{\frac{d}{dx}[x^8]\cdot x^3 - x^8\cdot\frac{d}{dx}[x^3]}{(x^3)^2}$

$$= \frac{8x^7\cdot x^3 - x^8\cdot 3x^2}{x^6}$$

$$= \frac{8x^{10} - 3x^{10}}{x^6} = \frac{5x^{10}}{x^6}$$

$$= 5x^4$$

Is this correct? Sure.

Easier $\frac{d}{dx}\left[\frac{x^8}{x^3}\right] = \frac{d}{dx}[x^5] = 5x^4$