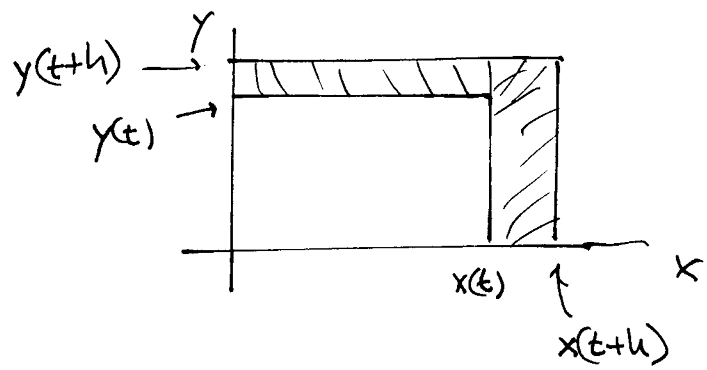
2.4 Product and Quotient Rules (cont'd)

Q: why does the product rule have the form that it does?

Recall: $\frac{df}{dt}$ represents the instantaneous rate of change.

Imagine a well-behaved wildfire

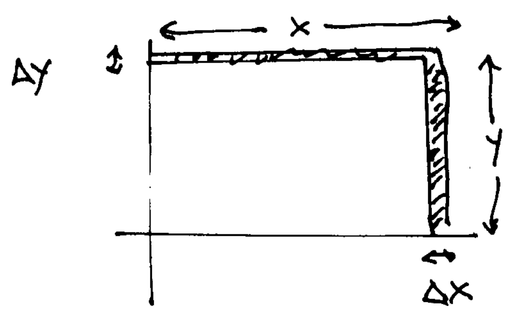


$A(t)$ = area burned as of time t

$A(t) = x(t) \cdot y(t)$

$$\text{avg rate of change} = \frac{\text{change in area}}{\text{change in time}} = \frac{\text{area of big rectangle} - \text{area of small rectangle}}{h}$$

$$A'(t) = \frac{dA}{dt} = \text{instantaneous rate of change} \approx \frac{\Delta x \cdot y + x \cdot \Delta y}{\Delta t}$$



$$\approx \frac{\Delta x}{\Delta t} \cdot y + x \cdot \frac{\Delta y}{\Delta t}$$

$$\rightarrow \frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt}$$

Quotient Rule: $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$

ex: $\dfrac{d}{dx}\left[\dfrac{2x^2}{x^2+1}\right] = \dfrac{\frac{d}{dx}[2x^2]\cdot(x^2+1) - 2x^2\cdot\frac{d}{dx}[x^2+1]}{(x^2+1)^2}$

$$= \frac{4x(x^2+1) - 2x^2\cdot 2x}{(x^2+1)^2}$$

$$= \frac{4x^3+4x-4x^3}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

38) $f(t) = \dfrac{t^2+1}{t^2-1}$

$$f'(t) = \frac{\frac{d}{dt}[t^2+1]\cdot(t^2-1) - (t^2+1)\cdot\frac{d}{dt}[t^2-1]}{(t^2-1)^2}$$

$$= \frac{2t\cdot(t^2-1) - (t^2+1)\cdot 2t}{(t^2-1)^2}$$

$$= \frac{2t^3-2t-2t^3-2t}{(t^2-1)^2} = \frac{-4t}{(t^2-1)^2}$$

Two special cases of the Quotient Rule:

1. $$\frac{d}{dx}\left[\frac{f(x)}{1}\right] = \frac{f'(x)\cdot 1 - f(x)\cdot \cancel{\frac{d}{dx}[1]}^{\,0}}{1^2} = \frac{f'(x)}{1} = f'(x)$$

2. $$\frac{d}{dx}\left[\frac{1}{g(x)}\right] = \frac{d}{dx}\left[(g(x))^{-1}\right] = \frac{\cancel{\frac{d}{dx}[1]}^{\,0}\cdot g(x) - 1\cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$= -\frac{g'(x)}{[g(x)]^2}$$

<u>Marginal Cost</u> or Revenue or Profit

$C(x)$ = what it <u>costs</u> to produce x items

$C'(x) = MC(x)$ = what it costs to go from x items to $x+1$ items

ex 2.3 #47) Answers: $C(x) = 24x^{2/3}$ so $C'(x) = 16x^{-1/3}$

a) $C'(8) = 16\cdot 8^{-1/3} = 8$ dollars/license

b) $C'(64) = 16\cdot 64^{-1/3} = 4$ dollars/license

2.5 Second and higher derivatives

ex: $f(x) = x^4 + 5x^3 + 2x^2 + 7x + 12$

$$\frac{df}{dx} = f'(x) = 4x^3 + 15x^2 + 4x + 7$$

$$\frac{d^2f}{dx^2} = f''(x) = 12x^2 + 30x + 4$$

$$\frac{d^3f}{dx^3} = f'''(x) = 24x + 30$$

$$f^{(4)}(x) = 24$$

$$f^{(5)}(x) = 0$$