Math 121 Thu 25 Feb 2016

2.5 Higher order derivatives

Another interpretation of derivatives

If $s(t)$ = distance or position, say t = time (seconds)
$s(t)$ = distance (meters)

then $s'(t) = \frac{ds}{dt}$ = velocity (like speed, but can be negative; units = meters/second)

and $s''(t) = \frac{d^2s}{dt^2}$ = acceleration (units: meters/(second)
$= \frac{d}{dt}[\text{velocity}]$ $= \frac{\text{meters/second}}{\text{second}}$)

[Aside: The meaning of $\frac{d^2s}{dt^2} = \frac{d}{dt} \cdot \frac{d}{dt} \cdot s$]

36) After t hours a car $s(t) = 60t + \frac{100}{t+3}$ miles from its start.

Find the velocity after 2 hours.

$$\text{velocity} = s'(t) = \frac{d}{dt}[60t] + \frac{d}{dt}\left[\frac{100}{t+3}\right]$$

$$= 60\frac{d}{dt}[t] + 100\frac{d}{dt}\left[\frac{1}{t+3}\right]$$

$$= 60 + 100\left[\frac{\cancel{\frac{d(1)}{dt}(t+3)} - 1 \cdot \frac{d}{dt}(t+3)}{(t+3)^2}\right]$$

$$= 60 + 100\left[\frac{-\frac{d}{dt}[t+3]}{(t+3)^2}\right] = 60 + 100\left[\frac{-1}{(t+3)^2}\right]$$

using:
$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

$$s'(t) = 60 - \frac{100}{(t+3)^2}$$

36 contd) So when $t = 2$ hours, the velocity is

$$s'(2) = 60 - \frac{100}{(2+3)^2} = 60 - \frac{100}{25}$$

$$\left.\frac{ds}{dt}\right|_{t=2} = 60 - 4 = 56 \text{ miles/hour}$$

38) A marble is dropped from the Sears Tower. Its height above the ground t seconds after it's dropped is $s(t) = 1454 - 16t^2$ feet.

a) How long does it take to reach the ground?

set $s(t) = 0$
height =

$$0 = 1454 - 16t^2 \qquad \text{Add } 16t^2 \text{ to both sides:}$$

$$16t^2 = 1454 \qquad \text{Divide by 16:}$$

$$t^2 = 90.875$$

$$t = \sqrt{90.875} = 9.53 \text{ seconds}$$

b) Find the velocity when the marble hits the ground.

$$\text{velocity} = s'(t) = -32t$$

When $t = 9.53$ s $\quad s'(9.53) = -32(9.53)$

$$= -305 \text{ ft/s}$$

c) Find the acceleration.

$$s''(t) = -32 \text{ ft/s}^2$$

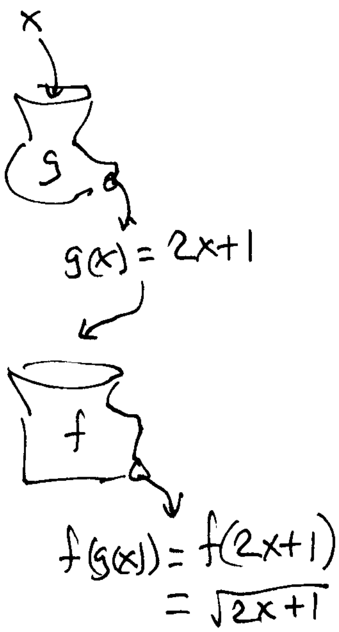
2.6 Chain Rule

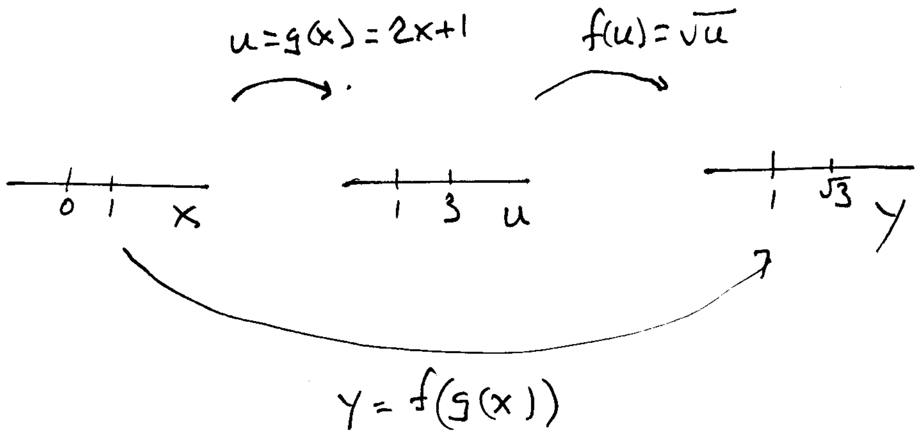
Recall the idea of composition of functions

ex: $f(x) = \sqrt{x}$ and $g(x) = 2x+1$

Find $f(g(x)) = f(2x+1)$

$= \sqrt{2x+1}$





ex: Find $f(x)$ and $g(x)$ so that $\sqrt{x^5 - 7x + 1}$ is the composite function $f(g(x))$.

Answer: Take $g(x) = x^5 - 7x + 1$ and $f(x) = \sqrt{x}$

Chain Rule: $\boxed{\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)}$

ex: Find $\frac{d}{dx}\sqrt{2x+1} = \frac{d}{dx}(2x+1)^{1/2}$

$f(x) = x^{1/2}$

$g(x) = 2x+1$

$$\frac{d}{dx}\left[(2x+1)^{1/2}\right] = \frac{1}{2}(2x+1)^{-1/2}\cdot\frac{d}{dx}[2x+1]$$

$$= \frac{1}{2}(2x+1)^{-1/2}\cdot(2)$$

$$= (2x+1)^{-1/2} = \frac{1}{\sqrt{2x+1}}$$ ← simplified

14) $g(x) = (3x^3 - x^2 + 1)^5$ Find $g'(x)$.

$$g'(x) = 5(3x^3 - x^2 + 1)^4 \cdot \frac{d}{dx}[3x^3 - x^2 + 1]$$

$$= 5(3x^3 - x^2 + 1)^4 (9x^2 - 2x)$$

18) $f(x) = \sqrt{x^6 + 3x - 1} = (x^6 + 3x - 1)^{1/2}$

$$f'(x) = \frac{d}{dx}\left[(x^6 + 3x - 1)^{1/2}\right]$$

$$= \frac{1}{2}(x^6 + 3x - 1)^{-1/2}\underbrace{\frac{d}{dx}[x^6 + 3x - 1]}_{(6x^5 + 3)}$$

$$= (x^6 + 3x - 1)^{-1/2}\left(3x^5 + \frac{3}{2}\right)$$

40) $g(z) = z^2(2z^3 - z + 5)^4$

$$g'(z) = \frac{d}{dz}[z^2] \cdot (2z^3 - z + 5)^4 + z^2 \cdot \frac{d}{dz}[(2z^3 - z + 5)^4]$$

product rule

$$= 2z\,(2z^3 - z + 5)^4 + z^2 \cdot 4(2z^3 - z + 5)^3 \underbrace{\frac{d}{dz}[2z^3 - z + 5]}_{(6z^2 - 1)}$$

$$= 2z(2z^3 - z + 5)^4 + 4z^2(2z^3 - z + 5)^3(6z^2 - 1)$$