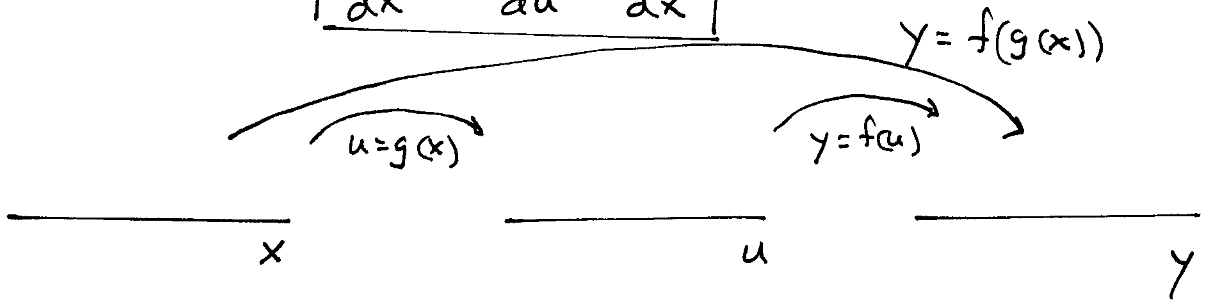
2.6 Chain Rule

... in Leibniz notation

If $y=f(u)$ and $u=g(x)$, then

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$



ex: $y = \sqrt[3]{u} = u^{1/3}$ and $u = x^3 - 4x$

Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(\frac{1}{3} u^{-\frac{2}{3}}\right) \cdot (3x^2-4)$$

Rewrite with x's only: $= \frac{1}{3}(x^3-4x)^{-2/3} \cdot (3x^2-4)$

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<u>examples</u> of chain rule in combination with other rules

<u>ex</u>. $f(x) = \left[(x^2+5x)^3 + 3x\right]^2$ Find $f'(x)$.

$$f'(x) = 2\left[(x^2+5x)^3 + 3x\right]^1 \cdot \frac{d}{dx}\left[(x^2+5x)^3+3x\right]$$

$$= 2\left[(x^2+5x)^3+3x\right]\left[\frac{d}{dx}\left[(x^2+5x)^3\right] + \frac{d}{dx}(3x)\right]$$

$$= 2\left[(x^2+5x)^3+3x\right]\left[3(x^2+5x)^2 \cdot \underbrace{\frac{d}{dx}\left[x^2+5x\right]}_{(2x+5)} + 3\right]$$

$$= 2\left[(x^2+5x)^3+3x\right]\left[3(x^2+5x)^2 \cdot (2x+5) + 3\right]$$

ex [chain rule twice] $y = \left(\sqrt[3]{x^4}\right)^{-1}$ Find $\frac{dy}{dx}$ the hard way

or $y = \left[(x^4)^{\frac{1}{3}}\right]^{-1}$

$$\frac{dy}{dx} = -\left[(x^4)^{1/3}\right]^{-2} \cdot \frac{d}{dx}\left[(x^4)^{1/3}\right] \quad \leftarrow \text{1st chain rule}$$

$$= -\left[(x^4)^{1/3}\right]^{-2} \cdot \frac{1}{3}(x^4)^{-2/3} \cdot \frac{d}{dx}\left[x^4\right] \quad \leftarrow \text{2nd use of chain rule}$$

$$= -\left[(x^4)^{1/3}\right]^{-2} \cdot \frac{1}{3}(x^4)^{-2/3} \cdot 4x^3$$

$$= -\frac{4}{3}\left[x^{4/3}\right]^{-2} \cdot (x^4)^{-2/3} \cdot x^3$$

$$= -\frac{4}{3} \cdot x^{-8/3} \cdot x^{-8/3} \cdot x^{3}$$

$$= -\frac{4}{3} \cdot x^{-\frac{8}{3}-\frac{8}{3}+3}$$

$$= -\frac{4}{3} x^{-7/3}$$

$$-\frac{8}{3} - \frac{8}{3} + \frac{9}{3} = \frac{-8-8+9}{3} = -\frac{7}{3}$$

<u>easy way</u>: $y = \left[(x^4)^{1/3}\right]^{-1} = x^{-4/3}$

by the power rule

$$\frac{dy}{dx} = -\frac{4}{3} \cdot x^{-\frac{4}{3}-1} = -\frac{4}{3} x^{-7/3}$$

<u>ex</u>: [outer: product rule; inner factor(s): chain rule]
Find $f'(x)$.

$$f(x) = x^5 (3x+2)^3$$

Product Rule

$$f'(x) = \frac{d}{dx}\left[x^5\right] \cdot (3x+2)^3 + x^5 \cdot \frac{d}{dx}\left[(3x+2)^3\right]$$

$$= 5x^4 \cdot (3x+2)^3 + x^5 \cdot 3(3x+2)^2 \cdot \underbrace{\frac{d}{dx}[3x+2]}_{3}$$

$$= 5x^4 (3x+2)^3 + 9x^5 (3x+2)^2$$