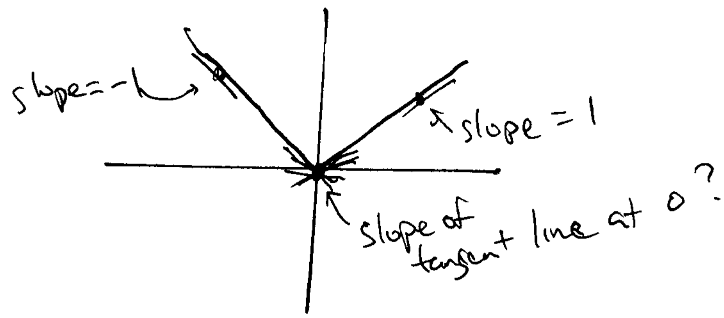
2.7 Nondifferentiable functions

ex: $f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$



$$f'(x) = \begin{cases} \frac{d}{dx}[x] & \text{if } x > 0 \\ \frac{d}{dx}[-x] & \text{if } x < 0 \end{cases} = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$

What about $f'(0)$? If this exists it will be

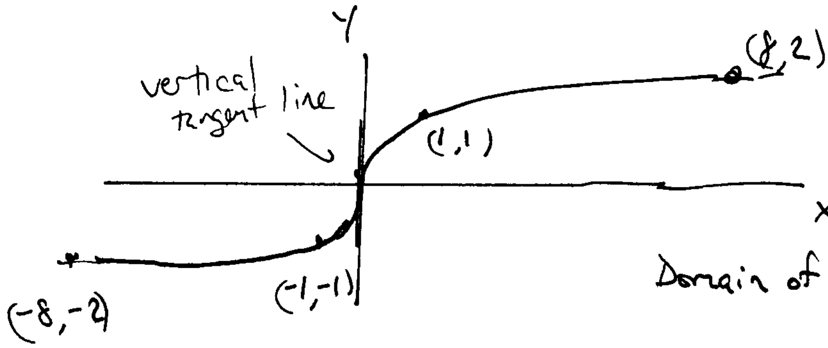
$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{|0+h| - |0|}{h} = \lim_{h\to 0} \frac{|h|}{h}$$

$$\frac{|h|}{h} = \begin{cases} \frac{h}{h} & \text{if } h > 0 \\ \text{undefined} & \text{if } h = 0 \\ \frac{-h}{h} & \text{if } h < 0 \end{cases} = \begin{cases} 1 & \text{if } h > 0 \\ \text{undefined} & \text{if } h = 0 \\ -1 & \text{if } h < 0 \end{cases}$$

$\lim_{h \to 0^+} \frac{|h|}{h} = \lim_{h\to 0} 1 = 1$ but $\lim_{h \to 0^-} \frac{|h|}{h} = \lim_{h \to 0} -1 = -1$

Since $1 \neq -1$ $\lim_{h\to 0} \frac{|h|}{h}$ does not exist. So $f'(0)$ does not exist.

ex: $f(x) = x^{1/3} = \sqrt[3]{x}$



Domain of $f = (-\infty, \infty)$

$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$ ← undefined when $x = 0$

But extremely positive when x is near 0.

Let $x = 10^{-6}$, then $x^2 = 10^{-12}$, $\sqrt[3]{x^2} = (10^{-12})^{1/3} = 10^{-4}$,

and $\frac{1}{\sqrt[3]{x^2}} = \frac{1}{10^{-4}} = 10^4$ and $\frac{1}{3\sqrt[3]{x^2}} = \frac{1}{3} \cdot 10^4$

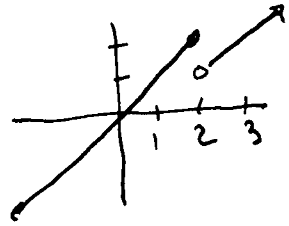
$\approx 3{,}333.\overline{3}$

$f'(0)$ does not exist.

Fact: If $f'(a)$ exists (that is, if f is differentiable at $x=a$) then f is continuous at $x=a$.

That is, differentiable implies continuous.

ex: $f(x) = \begin{cases} x & \text{if } x \leq 2 \\ x-1 & \text{if } x > 2 \end{cases}$



Since f is discontinuous at 2 it is also not differentiable at $x = 2$.

3.1 Graphing with 1st derivative

<u>Defn</u>: A <u>critical number</u> of a function f is an x-value in the <u>domain</u> of f where
either (i) $f'(x) = 0$ (horizontal tangent line)
or (ii) $f'(x)$ does not exist
(corner or vertical tangent line)

<u>Remark</u>: Critical numbers will supply the candidates for locations of relative maxima and minima.

6) Find the critical numbers of

$$f(x) = x^3 - 27x$$

$$0 = f'(x) = 3x^2 - 27$$

$$= 3(x^2 - 9) = 3(x-3)(x+3)$$

solns: $x = 3, -3$

Two critical numbers: $3, -3$

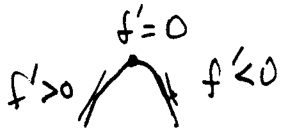
16) $f(x) = x^3 - x^2 + 15$

$$0 = f'(x) = 3x^2 - 2x = x(3x-2)$$

critical nos: $0, \quad 2/3$

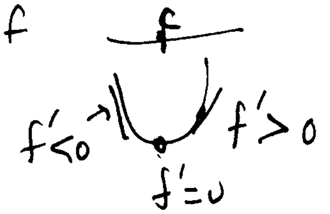
First derivative test:

If c is a critical number of function f



and f' goes from positive to negative then f has a relative max. at c.

If



f' goes from negative to positive at c then f has a rel. min at c.

20) Sketch the graph using f'.

$$f(x) = x^4 - 4x^3 - 8x^2 + 64$$

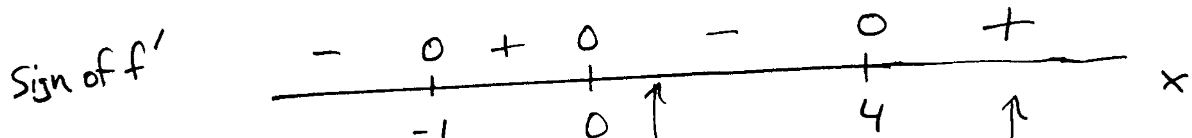
Find C.Ns:

$$0 = f'(x) = 4x^3 - 12x^2 - 16x$$

$$0 = 4x(x^2 - 3x - 4)$$

$$0 = 4x(x-4)(x+1)$$

CNs: $0, 4, -1$



Let $x=1$

$4(1)(1-4)(1+1)$

Let $x=5$

$4(5)(5-4)(5+1) > 0$

f is increasing on $(-1, 0)$ and $(4, \infty)$

f is decreasing on $(-\infty, -1)$ and $(0, 4)$

20 cnt'd)

x	$f(x)$
-1	61
0	64
4	-64