3.1 Graphing with 1st derivative (cont'd): Rational Functions

38) $f(x) = \dfrac{10x+30}{2x-6} = \dfrac{2(5x+15)}{2(x-3)} = \dfrac{5x+15}{x-3}$

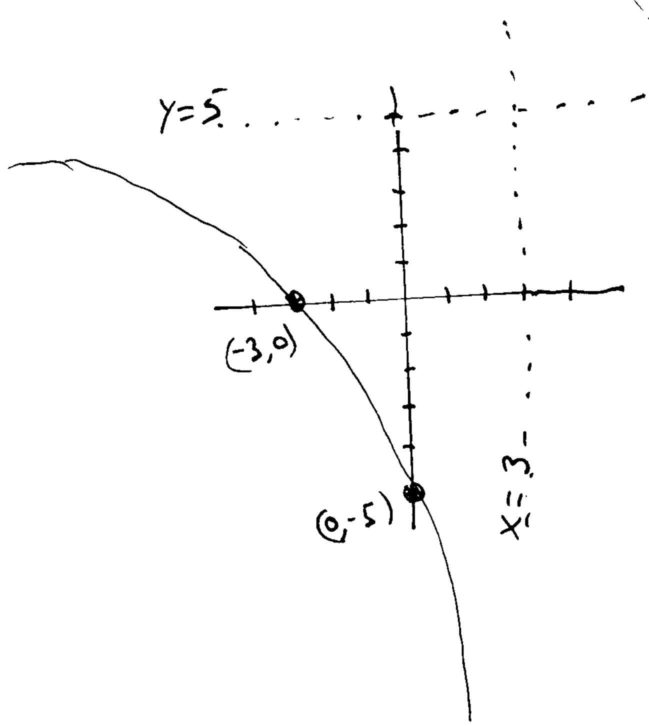
Domain? All reals except 3.

Vertical asymptote? The line $x=3$.
(that is, where the denominator is zero).

Horiz? The line $y=5$ [Reason: deg(top) = deg(bottom)
and $\dfrac{\text{leading coeff. of top}}{\text{leading bottom coeff}} = \dfrac{5}{1}$]

[or $\lim\limits_{x\to\infty} \dfrac{5x+15}{x-3} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}} = \lim\limits_{x\to\infty} \dfrac{5+\frac{15}{x}}{1-\frac{3}{x}}$

$= \dfrac{5+0}{1-0} = 5$]



x-intercept? Solve $0 = 5x+15$
$= 5(x+3)$
$\Rightarrow x = -3$

So $(-3,0)$ is the x-intercept

y-intercept? $f(0) = \dfrac{5(0)+15}{0-3} = -5$

$(0,-5)$ is y-int.

$$f'(x) = \frac{d}{dx}\left[\frac{5(x+3)}{x-3}\right] = 5\,\frac{d}{dx}\left[\frac{x+3}{x-3}\right]$$

$$= 5 \cdot \frac{\frac{d}{dx}[x+3]\cdot(x-3) - (x+3)\cdot\frac{d}{dx}[x-3]}{(x-3)^2}$$

$$= 5 \cdot \frac{(x-3) - (x+3)}{(x-3)^2} = 5 \cdot \frac{x-3-x-3}{(x-3)^2}$$

$$= 5 \cdot \frac{-6}{(x-3)^2} = \frac{-30}{(x-3)^2}$$

-30 ← always negative
$(x-3)^2$ ← always positive
} negative

sign of f': − | Not in domain | −

+3

conclusion: ↘ decreasing | ↘ decreasing

4c) Sketch the graph after making a sign diagram for f' and finding relative max/min and asymptotes.

$$f(x) = \frac{x^2-1}{x^2+1} = \frac{(x-1)(x+1)}{x^2+1}$$

Vertical asymptotes? None.
Horizontal " ? $y = 1$

x-intercepts? $(1,0)$, $(-1,0)$

y-intercept? $(0,-1)$

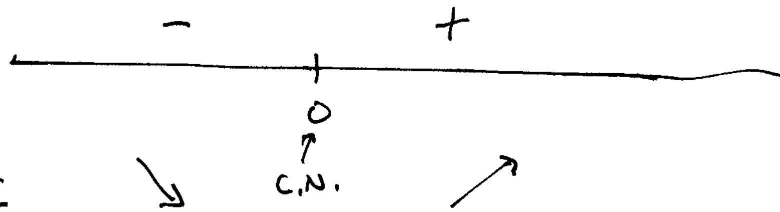
Critical nos?

$$f'(x) = \frac{2x\cdot(x^2+1) - (x^2-1)\cdot 2x}{(x^2+1)^2}$$

$$= \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2} \leftarrow \text{pos.}$$

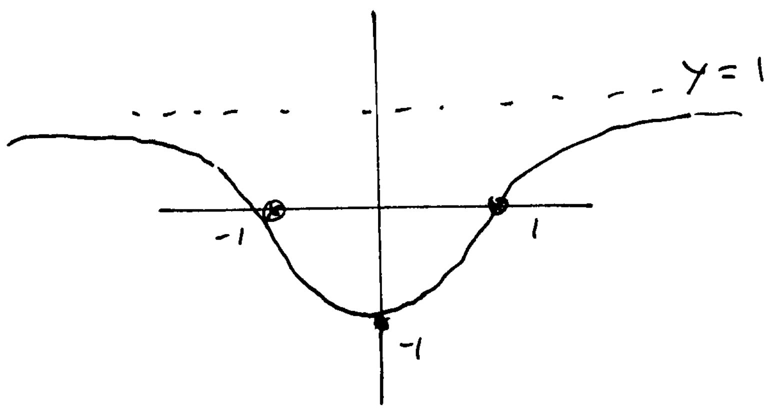
So C.Number is where $4x=0$ so $x=0$

sgn of f'



How to evaluate f' at a test pt: $f'(-1) = \frac{4(-1)}{((-1)^2+1)^2} = \frac{-4}{4} = -1$

By 1st deriv test there is a rel. <u>min</u> at $x=0$.



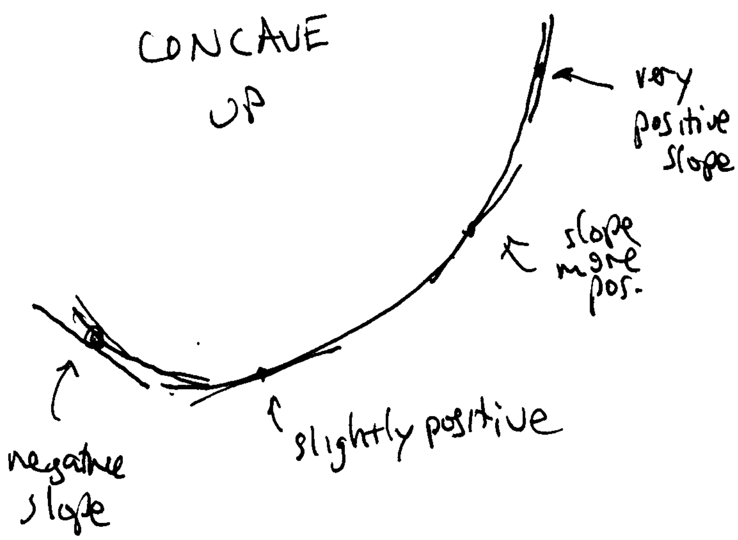
Intro to 3.2 What does the 2nd derivative tell us about a graph?

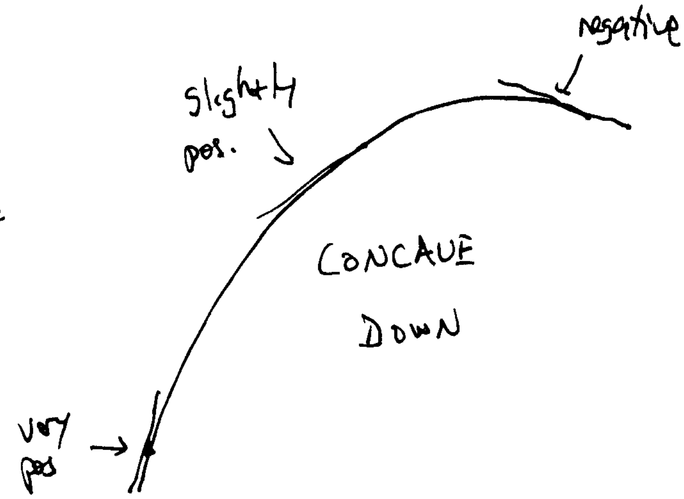
Interpretations of f'

1. Slope of tangent to graph
2. Instantaneous rate of change

Interpretation of f'':

by 1 and 2: Instantaneous rate of change of the slope of the tangent line.





$f'' > 0$

$$f'' = \frac{d}{dx}[\text{slope}] = \frac{d}{dx}\left[\frac{df}{dx}\right]$$

$f'(x)$ is an increasing function here.

$f'' < 0$

$$f'' = \frac{d}{dx}[\text{slope}] = \frac{d}{dx}\left[\frac{df}{dx}\right]$$

$f'(x)$ is a decreasing function here.