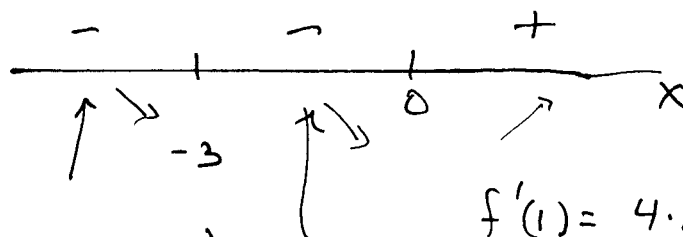


3.2 Graphing using first and second derivativewarmup: Find the critical numbers:

(1)
from #12) $f(x) = x^4 + 8x^3 + 18x^2 + 8$

$$\begin{aligned} 0 = f'(x) &= 4x^3 + 24x^2 + 36x \\ &= 4x(x^2 + 6x + 9) \\ &= 4x(x+3)^2 = 4x(x+3)(x+3) \end{aligned}$$

critical numbers: $0, -3, -3$ sign of $f'(x)$ 

test pts:

$$\begin{aligned} f'(-4) &= 4(-4)(-4+3)(-4+3) \\ &= (+)(-)(-)(-) \\ &< 0 \end{aligned}$$

$$\begin{aligned} f'(-1) &= 4(-1)(-1+3)(-1+3) \\ &= (+)(-)(+)(+) < 0 \end{aligned}$$

$$f'(1) = 4 \cdot 1(1+3)(1+3) > 0$$

Remark: Because -3 is a zero of $f'(x)$ of multiplicity 2
the signs of $f'(x)$ do NOT alternate.

warmup
(2):
from #36)

$$f(x) = 30x^{1/3} - 10x$$

$$= 30\sqrt[3]{x} - 10x$$

Domain of $f =$
 $(-\infty, \infty)$

$$f'(x) = 10x^{-2/3} - 10 = \frac{10}{\sqrt[3]{x^2}} - 10$$

$$= \frac{10}{\sqrt[3]{x^2}} - \frac{10\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \frac{10(1 - \sqrt[3]{x^2})}{\sqrt[3]{x^2}}$$

A fraction will be undefined where the denominator is zero.

A fraction will be zero where the numerator is zero.

i) For what x is $10(1 - \sqrt[3]{x^2}) = 0$?

$$1 - \sqrt[3]{x^2} = 0$$

$$1 = \sqrt[3]{x^2}$$

$$1^3 = (\sqrt[3]{x^2})^3$$

$$1 = x^2 \Rightarrow x = \pm\sqrt{1} = \pm 1$$

ii) For what x is $\sqrt[3]{x^2} = 0$?

$$(\sqrt[3]{x^2})^3 = 0^3$$

$$x^2 = 0$$

$$x = \pm\sqrt{0} = 0$$

Critical numbers: $x = -1$, $x = 0$, $x = 1$

We will come back to these questions.

Back to #12) $f'(x) = 4x^3 + 24x^2 + 36x$

$$f''(x) = 12x^2 + 48x + 36$$

where is $f''(x) = 0$?

$$0 = 12x^2 + 48x + 36$$

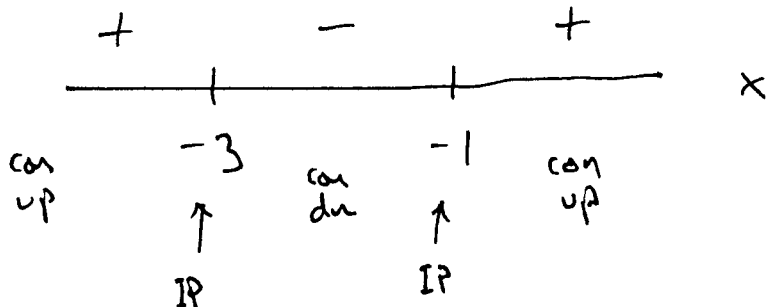
$$= 12(x^2 + 4x + 3)$$

$$= 12(x+1)(x+3)$$

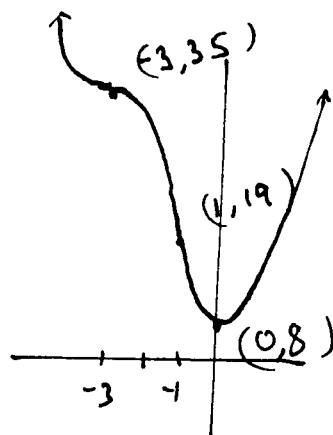
zeros:

$$-1, -3$$

sign of $f''(x)$



x	$f(x)$
-3	35
-1	19
0	8



(4)

Back to #36) $f'(x) = 10x^{-2/3} - 10$

$$f''(x) = -\frac{20}{3}x^{-5/3} = -\frac{20}{3} \cdot \frac{1}{\sqrt[3]{x^5}}$$

i) where is $f''(x) = 0$? No place.

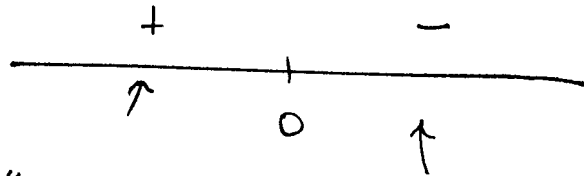
ii) where is $f''(x)$ undefined?

Solve $0 = \sqrt[3]{x^5}$

$$0 = 0^3 = x^5$$

so $x = \sqrt[5]{0} = 0$ Answer: $x = 0$.

sign $f''(x)$



$$\begin{aligned} f''(-1) &= -\frac{20}{3}(-1)^{-5/3} \\ &= \frac{20}{3} > 0 \end{aligned}$$

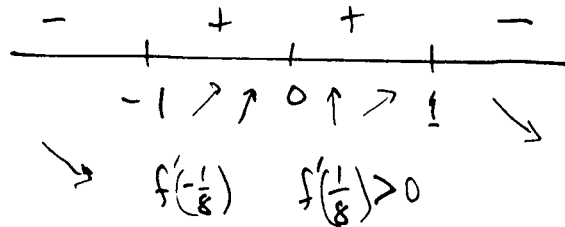
con up

$$f''(1) = -\frac{20}{3} \cdot 1^{-5/3} = -\frac{20}{3} < 0$$

con dn

sign of $f'(x)$

$$f'(x) = \frac{10(1 - x^{2/3})}{x^{2/3}}$$



$$f'(-\frac{1}{8}) \quad f'(\frac{1}{8}) > 0$$

$$f'(8) = \frac{10(1 - 8^{2/3})}{8^{2/3}}$$

$$= \frac{10(1 - 4)}{4} < 0$$

note: $x^{2/3} > 0$
for all x
for $x^{2/3} = \sqrt[3]{x^2} > 0$

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of 5

x	$f(x) = 30\sqrt{x} - 10x$
-1	-20
0	0
1	20

