3.2 #38) [Similar to Quiz 5 #2]

a) Do a sign diagram for f'

b) " " " " " f''

c) Sketch by hand, showing rel. max/min, inflection pts.

$$f(x) = 6x - 10x^{3/5} = 6x - 10\sqrt[5]{x^3}$$

Domain $= (-\infty, \infty)$

$$f'(x) = 6 - 6x^{-2/5}$$

i) where is $f'(x) = 0$?

$$0 = 6 - 6x^{-2/5}$$

$$0 = 6 - \frac{6}{\sqrt[5]{x^2}}$$

$$0 = \frac{6\sqrt[5]{x^2}}{\sqrt[5]{x^2}} - \frac{6}{\sqrt[5]{x^2}}$$

A fraction is zero exactly where the numerator is zero.

$$0 = \frac{6\left(\sqrt[5]{x^2} - 1\right)}{\sqrt[5]{x^2}}$$

$$0 = \sqrt[5]{x^2} - 1$$

$$\left(\sqrt[5]{x^2}\right)^5 = 1^5$$

$$x^2 = 1$$

$$x = \pm\sqrt{1} = \pm 1$$

ii) where is $f'(x)$ undefined?

$$f'(x) = 6 - 6x^{-2/5}$$

$$= \frac{6\left(\sqrt[5]{x^2} - 1\right)}{\sqrt[5]{x^2}}$$

A fraction is undefined where the denominator is zero.

So solve $\sqrt[5]{x^2} = 0$

$$\sqrt[5]{x^2}^{\,5} = 0^5$$

$$x^2 = 0$$

$$x = 0$$

C.Ns: $-1, 0, 1$

increase/decre ↗ ↘ ↘ ↗

sign of $f'(x)$: $+$ at $x<-1$, 0 at $x=-1$, $-$ between -1 and 0, undefined at $x=0$, $-$ between 0 and 1, 0 at $x=1$, $+$ at $x>1$

$$f'(x) = \frac{6\left(\sqrt[5]{x^2}-1\right)}{\sqrt[5]{x^2}}$$

always positive

let $x = \pm\frac{1}{2}$ so $x^2 = \frac{1}{4}$

$\sqrt[5]{\frac{1}{4}} < 1$ so $\sqrt[5]{\frac{1}{4}} - 1 < 0$

$$f'(x) = 6 - 6x^{-2/5}$$

$$f''(x) = \frac{12}{5}x^{-7/5} = \frac{12}{5\sqrt[5]{x^7}}$$

← undefined if $x=0$

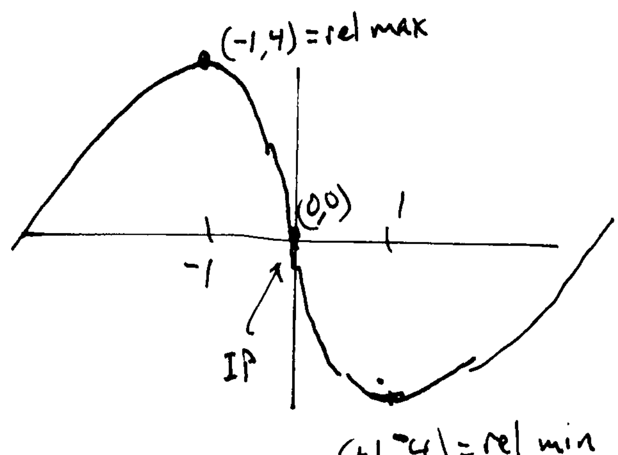
So possible inflection point at $x=0$

sign of $f''(x)$: $-$ for $x<0$, $+$ for $x>0$

concavity: con. dn (for $x<0$), con up (for $x>0$)

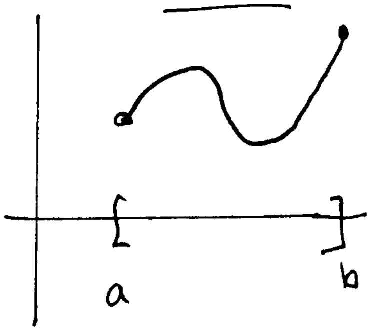
inflection pt at $x=0$

x	$f(x)$
-1	4
0	0
1	-4



3.3 Optimization

Absolute maxima and minima



Fact: If f is ~~defin~~ continuous on a closed interval $[a, b]$ then it will have an absolute max and min. These will occur either
i) at a critical number in (a, b)
OR ii) at the endpoints.

ex. $f(x) = x^3 - 27x$ on $[-2, \overset{4}{\cancel{x}}]$

Find the abs. max and min.

CNs? $0 = f'(x) = 3x^2 - 27$
$$= 3(x^2 - 9) = 3(x-3)(x+3)$$

critical nos: $x = 3$, ~~$x = -3$~~

x	$f(x)$	
-2	46	← absolute max on $[-2, 4]$
3	-54	← absolute min on $[-2, 4]$
4	-44	

32) Accident rate $= A(x) = \dfrac{x^2}{x^3+256}$

x = hundreds of hrs of flight

For what x is the rate maximized?

Critical nos?

$$0 = A'(x) = \frac{2x(x^3+256) - x^2(3x^2)}{(x^3+256)^2}$$

SOME SCRATCH WORK:

$2x(x^3+256) - 3x^4$

$= x[2(x^3+256) - 3x^3]$

$= x[2x^3+512-3x^3]$

$= x(-x^3+512)$

$$0 = \frac{2x(x^3+256) - 3x^4}{(x^3+256)^2}$$

$$0 = \frac{x[2(x^3+256) - 3x^3]}{(x^3+256)^2}$$

$$0 = \frac{x(-x^3+512)}{(x^3+256)^2}$$

multiply both sides by $(x^3+256)^2$: $\quad 0 = x(-x^3+512)$

$\cancel{x=0}$ or $0 = -x^3+512$

$x^3 = 512$

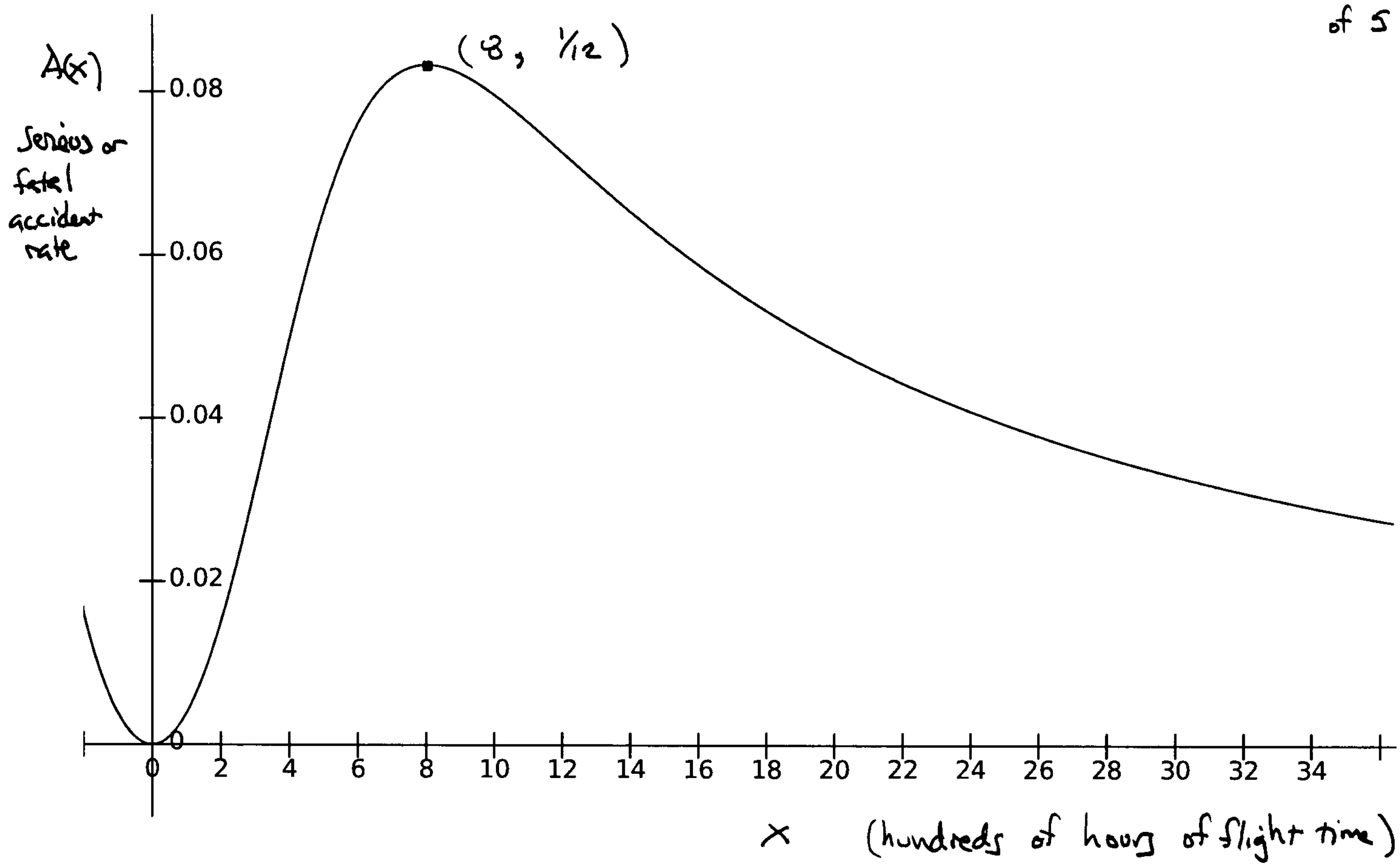
$x = \sqrt[3]{x^3} = \sqrt[3]{512} = (2^9)^{1/3}$

$= 2^3 = 8$ hundred hours

Is this a max or is it a min?

Second derivative test: $f''(8) < 0$ so rel max at $x = 8$

[Actually, the first derivative test is easier to apply.]



The likelihood of a pilot having a serious or fatal accident reaches its maximum when the pilot has flown 8 hundred hours.
(Perhaps before then, the pilot is cautious or supervised, then after then, the pilot gains judgement and competence.)