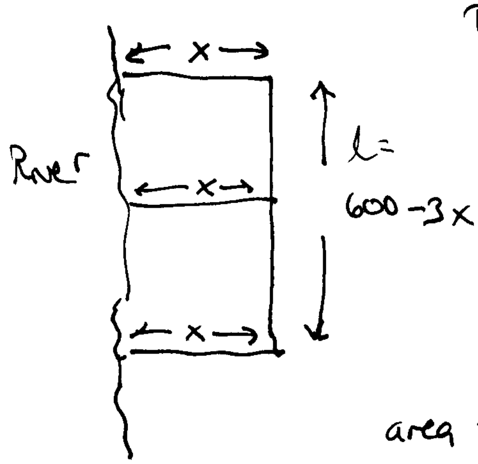
3.3 Optimization (cont'd)

How to come up with $f(x)$ to maximize/minimize

42)



Total amnt of fencing = 600 ft of fencing

total amt of fencing

$600 \text{ yds} = x + x + x + l$

$600 = 3x + l$

so

$600 - 3x = l$

Check:

$x + x + x + (600 - 3x) = 600 \text{ yds}$

$$\text{area} = f(x) = (\text{length})(\text{width})$$

$$= (600 - 3x)(x)$$

$$f(x) = -3x^2 + 600x$$

Domain of f? domain $= [0, 200]$

Critical numbers? $0 = f'(x) = -6x + 600$

$\Rightarrow 6x = 600 \Rightarrow x = 100 \text{ yds}$

$f''(x) = -6 < 0$ so rel max at $x = 100 \text{ yds}$

$\therefore$ Max occurs when $x = 100 \text{ yds}$

$600 - 3(100) = 300 \text{ yds}$

Maximum area $= f(100) = [600 - 3(100)](100)$

$= (300 \text{ yds})(100 \text{ yds})$

$= 30{,}000 \text{ yds}^2$ = maximized area

Dimensions:

of each enclosure: $100 \text{ yds} \times 150 \text{ yds}$

36) It costs \$70 to make each bicycle.
Fixed costs are \$100 per day.
Price function $= p(x) = 270 - 10x$

Q: How many bikes should be produced? At what price, to maximize profit? How much profit will be made?

$=$ the price you should choose (per bicycle) in order to sell x bicylces (in a day)

Task: Set the price to maximize profit.

$$\text{Profit} = P(x) = R(x) - C(x) = \text{revenue} - \text{cost}$$

$$R(x) = (\text{price per bike}) \cdot (\text{number of bikes})$$
$$= (p(x)) \cdot (x) = (270 - 10x)\, x$$
$$= 270x - 10x^2$$

That is, if we set the price at x dollars/bike we will take in $R(x) = 270x - 10x^2$ dollars/day

$$C(x) = \underbrace{100}_{\substack{\text{Fixed costs} \\ \text{(rent or whatever)}}} + \underbrace{70x}_{(70\ \text{dollars/bike})(x\ \text{bikes})} \quad \text{dollars each day}$$

$$\text{profit} = P(x) = R(x) - C(x)$$
$$= [270x - 10x^2] - [100 + 70x] = 270x - 10x^2 - 100 - 70x$$
$$= -10x^2 + 200x - 100$$

$$0 = \underset{\text{profit}}{\text{marginal}} = MP(x) = P'(x) = -20x + 200 \quad \Rightarrow \quad \text{critical number: } 20x = 200$$
$$x = 10 \text{ bikes}$$

$$\text{price} = p(10) = 270 - 10(10) = 270 - 100 = 170 \text{ dollars/bike}$$

$$\text{profit} = P(10) = -10(10)^2 + 200(10) - 100 = -1000 + 2000 - 100$$
$$= 900 \text{ dollar of profit}$$

Remark: $P(x) = R(x) - C(x)$

$$0 = P'(x) = R'(x) - C'(x)$$

So $R'(x) = C'(x)$

That is, $MR(x) = MC(x)$ when profit is maximized.