4.1 and 4.2 Exponential and Logarithmic Functions

Five ways to think of functions

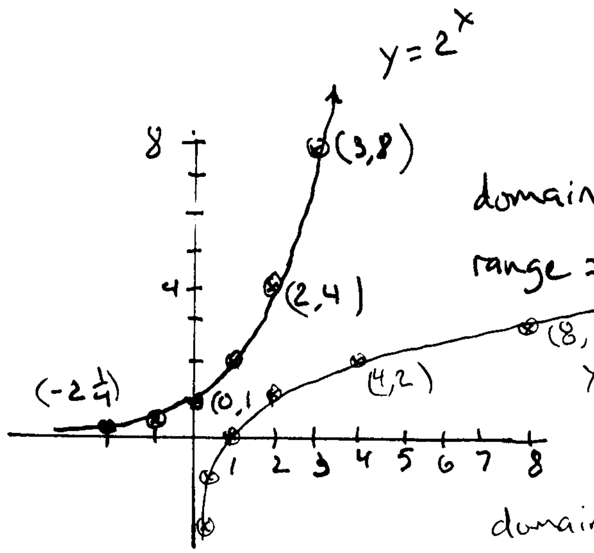
1. Machine
2. Equation
3. Table
4. Graph
5. Mapping

ex (our first exponential function)

$$f(x) = 2^x$$

x	2^x
-2	$1/4$
-1	$1/2$
0	1
$1/2$	$\sqrt{2}$
1	2
2	4
3	8
4	16

No repeats in 2nd column = one-to-one



domain = $(-\infty, \infty)$
range = $(0, \infty)$

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example: You have $1000 in an account which earns 8% per year compounded quarterly. How much do you have after 3 years.

Remark: 8% per year means 2% per quarter

$$= \frac{8\%}{4} = 0.02$$

After one quarter, you will have

$$1000 + .02 \cdot 1000$$
$$= 1000(1 + .02) = 1020 \text{ dollars}$$

After two quarters,

$$1020(1 + .02)$$
$$= 1000(1 + .02)(1 + .02)$$
$$= 1000(1.02)^2$$

After three quarters

$$1000(1.02)^3$$

After three years = 12 quarters

$$\text{balance} = 1000(1 + .02)^{4\cdot 3} = 1000(1.02)^{12}$$
$$= 1000\left(1 + \frac{.08}{4}\right)^{4\cdot 3} = \$1268.24$$

$$A = \boxed{\begin{pmatrix}\text{value after} \\ t \text{ years}\end{pmatrix} = P \cdot \left(1 + \frac{r}{m}\right)^{mt}}$$

Compound interest

P = principal = present value
r = interest rate
m = periods per years
t = number of years

If we solve for P:

$$\boxed{P = \frac{A}{\left(1+\frac{r}{m}\right)^{mt}}}$$

A = future value
P = present value

19) A = \$1,000,000 = future value
$r = 8\%$ m = 4 per year
$t = 60$ years
what P = present value?

$$P = \frac{1{,}000{,}000}{\left(1+\frac{.08}{4}\right)^{4\cdot 60}} = \frac{1{,}000{,}000}{(1.02)^{240}}$$

$$= \frac{1{,}000{,}000}{115.888}$$

$$= \$8{,}628.97$$

Continuous compounding

$$\boxed{A = \begin{pmatrix}\text{value after}\\ t \text{ years}\end{pmatrix} = Pe^{rt}}$$

P = present value
r = rate
t = time (years)
A = future value

Solve for P:

$$\boxed{P = \frac{A}{e^{rt}} = Ae^{-rt}}$$

Logarithms

$\log_2 8 = 3$ means $2^3 = 8$

Take the function $\log_2 x$

we input 8 and get 3 as output

Take the function 2^x

we input 3 and get 8 as output

Similarly

$\log x = \log_{10} x = y$ means $10^y = x$

OR

$\ln x = \log_e x = y$ means $e^y = x$

Six Properties of ln

(1) $\ln e^x = x$
(2) $e^{\ln x} = x$
$\Big\}$ Inverse properties

(3) $\ln(M \cdot N) = \ln M + \ln N$ Product rule of ln

(4) $\ln(M/N) = \ln M - \ln N$ Quotient " "

(5) $\ln(M^p) = p \ln M$ Power " "

(6) $\log_b M = \dfrac{\ln M}{\ln b}$ Change-of-base formula

ex: $\log_2 32 = \dfrac{\ln 32}{\ln 2} = \dfrac{3.466}{0.693} = 5$