Math 121 Tue, 19 April 2016

4.1 - .2 Exp and Log Functions (cont'd)

Remark: ① we can express any exp. function in terms of base e,

② we can express any log function in terms of ln.

ex: ① $f(x) = 2^x \underset{\substack{\uparrow \\ \text{property ②}}}{=} e^{\ln 2^x} \underset{\substack{\uparrow \\ \text{property ⑤}}}{=} e^{x \ln 2} = e^{(\ln 2)x}$

for exaple: By calculator $e^{(\ln 2)\cdot 5}$ should equal $2^5 = 32$

ex ② of remark : $g(x) = \log_5 x = \dfrac{\ln x}{\ln 5}$

For exaple, by calculator $g(125) = \log_5 125$

$$= \frac{\ln 125}{\ln 5} = 3$$

examples: [Applying properties of ln]

4.2 6) $f(x) = \ln\left(\frac{x}{2}\right) + \ln 2 = \ln\left[\left(\frac{x}{2}\right)\cdot 2\right] = \ln x$

12) $f(x) = \ln(e^{-2x}) + 3x + \ln 1$

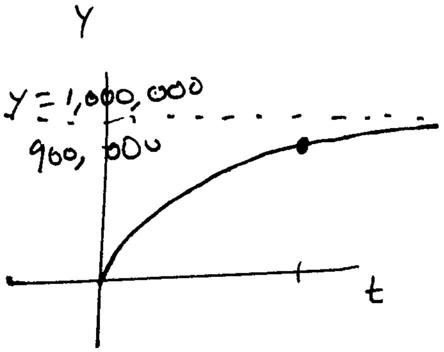
$= -2x + 3x + \ln e^{0} \rightarrow 0$

$= x$

§4.2 28) $f(t) = 1,000,000\,(1 - e^{-0.4t})$

t = time (hours) after news is announced

$f(t)$ = number of people in a city of a million who have heard the news



When will 900,000 have heard the news?

$$900,000 = 1,000,000\,(1 - e^{-0.4t})$$

$$\frac{900,000}{1,000,000} = \frac{\cancel{1,000,000}\,(1 - e^{-0.4t})}{\cancel{1000000}}$$

$$0.9 = 1 - e^{-0.4t}$$

$$-1 \qquad -1$$

$$-0.1 = -e^{-0.4t}$$ multiply by −1:

$$0.1 = e^{-0.4t}$$

$$\ln 0.1 = \ln e^{-0.4t}$$

$$\ln 0.1 = -0.4t$$

$$t = \frac{\ln 0.1}{-0.4} = 5.76 \text{ hours}$$

4.3 Derivatives of Log and Exponential Functions

$$\boxed{\frac{d}{dx}\ln x = \frac{1}{x}}$$

Derivative of the natural log function

ex: For $f(x) = x^2 \ln x$, find $f'(x)$.

$$f'(x) = \frac{d}{dx}[x^2]\cdot \ln x + x^2 \cdot \frac{d}{dx}[\ln x]$$

$$= 2x\ln x + x^2 \cdot \frac{1}{x}$$

$$= 2x\ln x + x$$

ex: $f(x) = \ln(x^2+1) = \ln(\text{stuff})$

$$f'(x) = \frac{1}{(\text{stuff})}\cdot\frac{d}{dx}[\text{stuff}]$$

← Chain Rule

$$= \frac{1}{x^2+1}\cdot\frac{d}{dx}[x^2+1]$$

$$= \frac{1}{x^2+1}\cdot 2x = \frac{2x}{x^2+1}$$

$$\boxed{\frac{d}{dx}\ln[g(x)] = \frac{1}{g(x)}\cdot\frac{d}{dx}[g(x)] = \frac{g'(x)}{g(x)}}$$

Chain Rule where the "outer function" is ln.

Recall the Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x))\cdot g'(x)$

4) $f(x) = \ln(x^3+1)$

$$f'(x) = \frac{1}{x^3+1} \cdot \frac{d}{dx}[x^3+1] = \frac{3x^2}{x^3+1}$$

$$= \frac{1}{x^3+1} \cdot 3x^2$$

ex: $f(x) = \ln[(x+3)(x^2+1)]$ Find $f'(x)$.

$$= \ln(x+3) + \ln(x^2+1) \quad \leftarrow \text{Recommended}$$

So $f'(x) = \frac{1}{x+3} \cdot \frac{d}{dx}[x+3] + \frac{1}{x^2+1} \cdot \frac{d}{dx}[x^2+1]$

$$= \frac{1}{x+3} \cdot 1 + \frac{1}{x^2+1} \cdot 2x$$

$$= \frac{1}{x+3} + \frac{2x}{x^2+1}$$

10) $f(x) = \ln(5x)$ The hard way:

$$f'(x) = \frac{1}{5x} \cdot \frac{d}{dx}[5x] = \frac{1}{5x} \cdot 5 = \frac{5}{5x} = \frac{1}{x}$$

The easy way

$$f(x) = \ln(5x) = \ln 5 + \ln x$$

$$f'(x) = \frac{d}{dx}[\ln 5] + \frac{d}{dx}[\ln x] = 0 + \frac{1}{x} = \frac{1}{x}$$

$$\boxed{\frac{d}{dx} e^x = e^x}$$

Derivative of the exponential function

ex: $\frac{d}{dx} e^{x^3} = e^{x^3} \cdot \frac{d}{dx}[x^3] = e^{x^3} \cdot 3x^2$

$= 3x^2 e^{x^3}$

$$\boxed{\begin{aligned} \frac{d}{dx}\left[e^{g(x)}\right] &= e^{g(x)} \cdot \frac{d}{dx}[g(x)] \\ &= g'(x) \cdot e^{g(x)} \end{aligned}}$$

Chain Rule where the "outer" function is exp function.

ex: $\frac{d}{dx} e^{5x} = e^{5x} \cdot \frac{d}{dx}[5x] = 5e^{5x}$

$$\boxed{\frac{d}{dx} e^{kx} = ke^{kx}}$$

12) $f(x) = x^3 e^x$ By Product Rule

$f'(x) = \frac{d}{dx}[x^3] \cdot e^x + x^3 \cdot \frac{d}{dx}[e^x]$

$= 3x^2 e^x + x^3 e^x$

22) $f(x) = \ln(e^x + e^{-x})$

$f'(x) = \frac{1}{e^x + e^{-x}} \cdot \frac{d}{dx}[e^x + e^{-x}]$

$= \frac{1}{e^x + e^{-x}} \cdot [e^x + (-1)e^{-x}] = \frac{e^x - e^{-x}}{e^x + e^{-x}}$