

4.3 $\frac{d}{dx} e^x$ and $\frac{d}{dx} \ln x$ [cont'd]

(1)
of 2

20) $f(x) = \ln e^x = x$
 $\uparrow$ easy way
 So $f'(x) = 1$.

Hard way:

$$f'(x) = \frac{1}{e^x} \cdot \frac{d}{dx} [e^x] = \frac{e^x}{e^x} = 1$$

18) $f(x) = x \ln x - x$

$$\begin{aligned} f'(x) &= \frac{d}{dx} [x \ln x] - \frac{d}{dx} [x] \\ &= \frac{d}{dx} [x] \cdot \ln x + x \cdot \frac{d}{dx} [\ln x] - 1 \\ &= 1 \cdot \ln x + x \cdot \frac{1}{x} - 1 \\ &= \ln x + 1 - 1 = \ln x \end{aligned}$$

26) $f(x) = \sqrt{e} = e^{1/2}$, $f'(x) = 0$

24) $f(x) = e^x$, $f'(x) = e \approx 2.718281828$

Similar to:

$g(x) = 3x$ $g'(x) = 3$

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of 2

$$28) \quad f(x) = x^2 e^x - 2 \ln x + (x^2 + 1)^3$$

$$f'(x) = \frac{d}{dx}[x^2] \cdot e^x + x^2 \cdot \frac{d}{dx}[e^x] - 2 \cdot \frac{1}{x} + 3(x^2 + 1)^2 \cdot \frac{d}{dx}[x^2 + 1]$$

$$= \underbrace{2xe^x + x^2 e^x}_{\text{optional: } (2x + x^2)e^x} - \frac{2}{x} + 6x(x^2 + 1)^2$$

34)

$$\begin{aligned} f(x) &= \ln \frac{1}{e^{x^2}} \\ &= \ln e^{-x^2} \\ &= -x^2 \end{aligned}$$

what is the easy way?

by properties of exponents

$$f'(x) = -2x$$

$$38) \quad f(t) = (2t + \ln t)^3$$

Chain Rule

$$\begin{aligned} f'(t) &= 3(2t + \ln t)^2 \cdot \frac{d}{dt}[2t + \ln t] \\ &= 3(2t + \ln t)^2 \cdot \left(2 + \frac{1}{t}\right) \end{aligned}$$

Try this: 36) $f(t) = \sqrt{e^{2t} + 4} = (e^{2t} + 4)^{1/2}$

$$\begin{aligned} \text{ANSWER: } f'(t) &= \frac{1}{2}(e^{2t} + 4)^{-1/2} \cdot \frac{d}{dt}[e^{2t} + 4] \\ &= \frac{1}{2}(e^{2t} + 4)^{-1/2} \cdot 2e^{2t} \end{aligned}$$

$$= \frac{e^{2t}}{\sqrt{e^{2t} + 4}}$$