Math 121 - Tue 26 April 2016

4.3

82) A \$10,000 car depreciates so that its value after t years is

$$V(t) = 10{,}000\, e^{-0.35t}$$

$$V'(t) = 10{,}000 \cdot \frac{d}{dt}\left[e^{-0.35t}\right]$$

$$= 10{,}000 \cdot \left(-0.35\, e^{-0.35t}\right)$$

$$V'(t) = -3{,}500\, e^{-0.35t} \text{ dollars/year}$$

a) what is the instantaneous rate of change of the value of the car when it is new ($t=0$)?

$$V'(0) = -3{,}500\, e^{-0.35(0)} = -3{,}500\, e^{0}$$

$$= -3{,}500\,(1) = -3{,}500 \text{ dollars/year.}$$

b) What is the rate of change after 2 years?

$$V'(2) = -3{,}500\, e^{-0.35(2)} = -3{,}500\, e^{-0.7}$$

$$= -3{,}500\,(0.49659) = -1{,}738 \text{ dollars/year}$$

4.4 Relative Rates of Change

Idea: If shoes prices are increasing at 3 dollars/year and shoes are priced at 60 dollars, the relative rate of change of price is

$$\frac{3 \cancel{\text{dollars}}/\text{years}}{60 \cancel{\text{dollars}}} = \frac{1}{20} \text{ per year}$$
$$= 0.05 \text{ year}^{-1}$$
$$= 5\% \text{ per year}$$

Similarly: A \$15,000 is increasing in price at \$300 per year.

The relative rate of increase is

$$\frac{300 \cancel{\text{dollars}}/\text{year}}{15{,}000 \cancel{\text{dollars}}} = \frac{3}{150} \text{ years}^{-1}$$
$$= \frac{1}{50} \text{ years}^{-1}$$
$$= 0.02 \text{ per year}$$
$$= 2\% \text{ per year}$$

Defn: $$\left(\begin{array}{l}\text{Relative rate} \\ \text{of change of } f(t)\end{array}\right) = \frac{f'(t)}{f(t)} = \frac{d}{dt} \ln f(t)$$

ex: [Back to §4.3 #82]

For the depreciating car $V(t) = 10{,}000\, e^{-0.35t}$

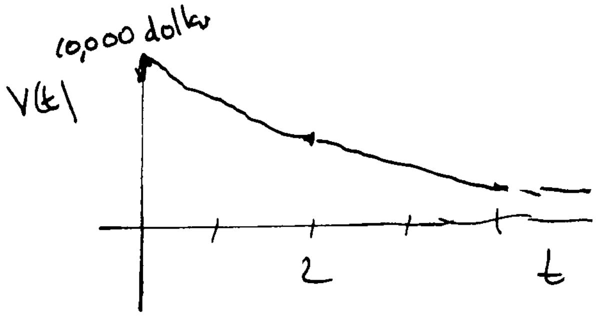
What is the relative rate change of value of the car (and what units)?

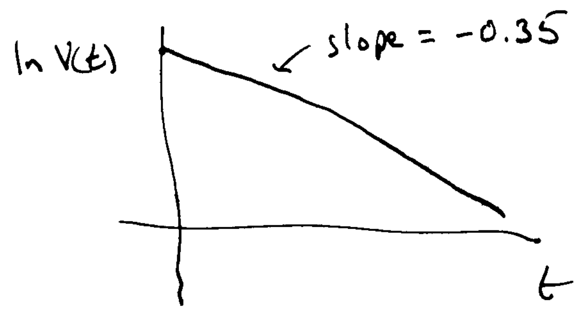
Method 1:
$$\frac{V'(t)}{V(t)} = \frac{-3{,}500\, \cancel{e^{-0.35t}}}{10{,}000\, \cancel{e^{-0.35t}}} = -\frac{3{,}500}{10{,}000}$$
a constant!

$$= -0.35 \text{ per year}$$
$$= -35\% \text{ per year}$$

Method 2: Use $\frac{d}{dt} \ln V(t)$.

$$V(t) = 10{,}000\, e^{-0.35t}$$
$$\ln V(t) = \ln\left(10{,}000\, e^{-0.35t}\right)$$
$$\ln V(t) = \ln 10{,}000 + \ln e^{-0.35t}$$
$$\ln V(t) = \ln 10{,}000 - 0.35t$$
$$\frac{d}{dt} \ln V(t) = -0.35 \text{ per year} = -35\% \text{ per year}$$





4.4 #14) $N(t)$ = national debt of a country (trillions of dollars)

t = years from now

$$N(t) = 0.4 + 1.2\, e^{0.01t}$$

Find the relative rate of change of the debt 10 years from now.

(method 2) $\ln N(t) = \ln\left(0.4 + 1.2\, e^{0.01t}\right)$

$$\text{relative rate of change} = \frac{d}{dt} \ln N(t) = \frac{d}{dt} \ln\left(0.4 + 1.2\, e^{0.01t}\right)$$

$$= \frac{1}{0.4 + 1.2\, e^{0.01t}} \cdot \left(0 + 1.2 \cdot 0.01\, e^{0.01t}\right)$$

$$= \frac{0.012\, e^{0.01t}}{0.4 + 1.2\, e^{0.01t}}$$

10 years from now

$t = 10$ so $\left(\text{rel. rate of change}\right) = \dfrac{0.012\, e^{0.01(10)}}{0.4 + 1.2\, e^{0.01(10)}} = \dfrac{0.012\, e^{0.1}}{0.4 + 1.2\, e^{0.1}}$

$= 0.00768$ per year

$= 0.768\%$ per year