5.1 Antiderivatives and indefinite integrals

ex: $3x^2$ is the derivative of what $f(x)$?

One answer: $f(x) = x^3$ because $f'(x) = 3x^2$

Another answer: $f(x) = x^3 + 1$ because $f'(x) = 3x^2$

Yet another: $f(x) = x^3 - 17$ because $f'(x) = 3x^2$.

These are all "antiderivatives" of $3x^2$.

Notation: $\int 3x^2\,dx = x^3 + C$ is the "indefinite integral" of $3x^2$.

ex: $\int 10x^4\,dx = 2x^5 + C$

check: $\frac{d}{dx}\left[2x^5 + C\right] = 2\cdot 5x^4 + 0 = 10x^4$

ex: $\int x^7\,dx = \frac{1}{8}x^8 + C$

check: $\frac{d}{dx}\left[\frac{1}{8}x^8\right] = \frac{1}{8}\cdot 8x^7 = x^7$.

What's going on? $\frac{d}{dx} x^8 = 8\cdot x^7$

$$\int x^7\,dx = \frac{1}{7+1}x^{7+1} + C = \frac{1}{8}x^8 + C$$

$$\boxed{\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C}$$

Power Rule for integration

ex: $\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx$

$$= \frac{1}{4/3} x^{4/3} + C = \frac{3}{4} x^{4/3} + C$$

check: $\frac{d}{dx}\left[\frac{3}{4} x^{4/3} + C\right] = \frac{3}{4} \cdot \frac{4}{3} x^{1/3} = \sqrt[3]{x}$

ex: $\int 10x^2\,dx = 10 \int x^2\,dx$

$$= 10 \cdot \frac{1}{3} x^3 + C = \frac{10}{3} x^3 + C$$

check: $\frac{d}{dx}\left[\frac{10}{3} x^3 + C\right] = \frac{10}{3} \cdot \frac{d}{dx} x^3 = \frac{10}{3} \cdot 3x^2 = 10x^2$

$$\boxed{\int k \cdot f(x)\,dx = k \cdot \int f(x)\,dx}$$

constant multiple rule for integration

ex: $\int (7x^6 + 6x^2)\,dx = \int 7x^6\,dx + \int 6x^2\,dx$

$$= 7\int x^6\,dx + 6\int x^2\,dx$$

$$= 7 \cdot \frac{1}{7} x^7 + 6 \cdot \frac{1}{3} x^3 + C$$

$$= x^7 + 2x^3 + C$$

Sum Rule for integration

$$\boxed{\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx}$$

Notation: $\int dx = \int 1 \cdot dx = \int x^0 \, dx$

$= \frac{1}{1} x^1 + C = x + C$

ex: $\int 17 \, dx = 17 \int dx = 17x + C$

$$\boxed{\int k \, dx = kx + C}$$ Integral of a constant

Remark: we can find the indefinite integral of any polynomial.

ex: $\int (4x^3 + 9x^2 + 8x + 17) \, dx$

$= \int 4x^3 dx + \int 9x^2 dx + \int 8x \, dx + \int 17 dx$

$= 4 \int x^3 dx + 9 \int x^2 dx + 8 \int x \, dx + 17 \int dx$

$= 4 \cdot \frac{1}{4} x^4 + 9 \cdot \frac{1}{3} x^3 + 8 \cdot \frac{1}{2} x^2 + 17x + C$

$= x^4 + 3x^3 + 4x^2 + 17x + C$

<u>Warning</u>: (1) There is <u>no</u> <u>product</u> <u>rule</u> ~~for~~ for integration.

(2) There is <u>no</u> <u>quotient</u> <u>rule</u> for integration.

<u>But</u> we may be able to get around this with a little algebra.

<u>ex</u>: $\int (x+3)^2 x\,dx = \int (x^2+6x+9)x\,dx$

$= \int (x^3+6x^2+9x)\,dx$

$= \frac{1}{4}x^4 + 2x^3 + \frac{9}{2}x^2 + C$

<u>ex</u>: $\int \frac{x^3+x}{\sqrt{x}}\,dx = \int \frac{x^3+x}{x^{1/2}}\,dx$

$= \int \left(\frac{x^3}{x^{1/2}} + \frac{x^1}{x^{1/2}}\right) dx = \int (x^{5/2} + x^{+1/2})\,dx$

$= \frac{2}{7}x^{7/2} + \frac{2}{3}x^{3/2} + C$

5.1 44) A company's marginal cost function is

$$MC(x) = 21x^{4/3} - 6x^{1/2} + 50$$

where x = number of units and fixed costs are \$3000, Find the cost function.

$$C(x) = \int MC(x)\,dx$$
$$= \int (21x^{4/3} - 6x^{1/2} + 50)\,dx$$
$$= 21 \cdot \frac{3}{7} x^{7/3} - 6 \cdot \frac{2}{3} x^{3/2} + 50x + C$$
$$= 9x^{7/3} - 4x^{3/2} + 50x + C$$

What is C?

$$\$3000 = C(0) = 9 \cdot 0^{7/3} - 4 \cdot 0^{3/2} + 50 \cdot 0 + C = C$$

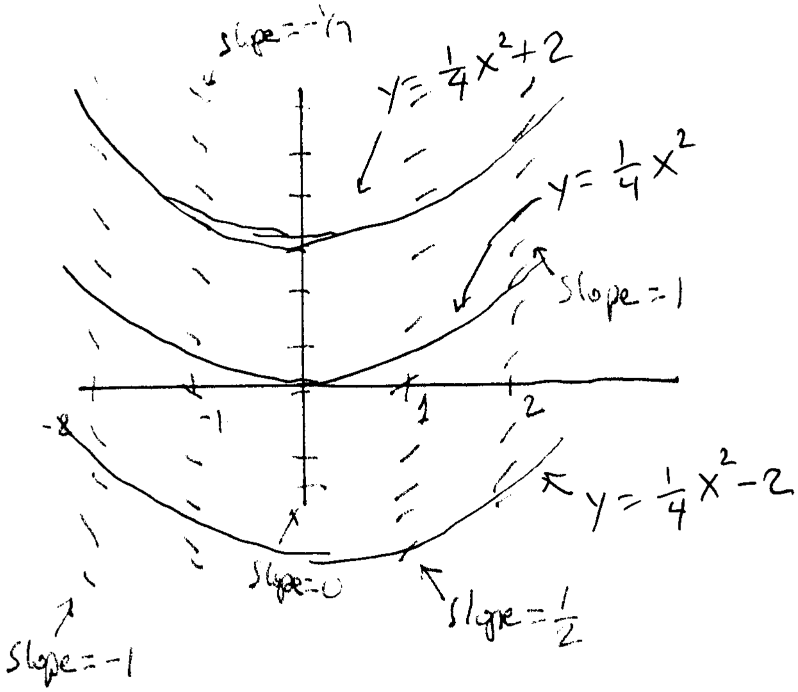
so $C = 3000$

$$C(x) = 9x^{7/3} - 4x^{3/2} + 50x + 3000$$

Graphical interpretation of indefinite integrals

ex: $\int \frac{1}{2} x \, dx$ means find $F(x)$ so that

$F'(x) = \frac{1}{2}x$ $\left(\text{so } F(x) = \frac{1}{4}x^2 + C \right)$



Draw line segments to represent slopes of potential tangent lines

The curves of the form

$y = \frac{1}{4}x^2 + C$

have tangent line (segments) with the specified slopes.