5.2 Integration using Log. and Exp. Functions

Basic integral formulas

example

① $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$ $\qquad$ $\int x^3 dx = \frac{1}{4}x^4 + C$

② $\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$ $\qquad$ $\int e^{3x} dx = \frac{1}{3} e^{3x} + C$

③ $\int \frac{1}{x} \, dx = \int x^{-1} dx = \ln|x| + C$

Plus: the sum rule and the constant multiple rule.

Why ②? We know $\frac{d}{dx}\left[e^{3x}\right] = 3e^{3x}$

So we can check that $\int e^{3x} dx = \frac{1}{3}e^{3x} + C$

as follows: $\frac{d}{dx}\left[\frac{1}{3}e^{3x} + C\right] = \frac{1}{3} \cdot \frac{d}{dx} e^{3x} + \frac{d}{dx} C$

$$= \frac{1}{3} \cdot 3e^{3x} + 0 = e^{3x}$$

ex: 6) $\int e^{0.02x} \, dx = \frac{1}{0.02} e^{0.02x} + C$

$$= \frac{1}{1/50} e^{0.02x} + C$$

$$= 50\, e^{0.02x} + C$$

4) $\int e^{x/3}\,dx = \int e^{\frac{1}{3}x}\,dx$

$= \frac{1}{1/3} e^{x/3} + C = 3e^{x/3} + C$

ex. $\int \frac{e^x + 1}{e^{2x}}\,dx = \int \left(\frac{e^x}{e^{2x}} + \frac{1}{e^{2x}}\right)dx$

$= \int \left(e^{x-2x} + e^{-2x}\right)dx = \int \left(e^{-x} + e^{-2x}\right)dx$

$= \int e^{-x}\,dx + \int e^{-2x}\,dx$

$= \frac{1}{-1} e^{-x} + \frac{1}{-2} e^{-2x} + C = -e^{-x} - \frac{1}{2} e^{-2x} + C$

ex: $\int \left(\frac{1}{x^2} + \sqrt{x} + e^{6x}\right)dx = \int \left(x^{-2} + x^{1/2} + e^{6x}\right)dx$

$= -x^{-1} + \frac{2}{3}x^{3/2} + \frac{1}{6}e^{6x} + C$

Remark: (1) $\int x^{-3}\,dx = \frac{1}{-2} x^{-2} + C$

(2) $\int x^{-2}\,dx = \frac{1}{-1} x^{-1} + C$

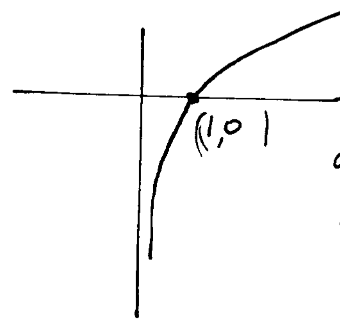
(3) $\int x^{-1}\,dx$ is NOT $\frac{1}{-0} x^{0} + C$

Rather $\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$

<u>Remark</u>: In the formula $\boxed{\int \frac{1}{x}\,dx = \ln|x| + C}$

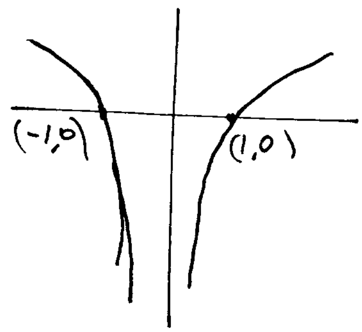
what's with the absolute value?

Think of the <u>function</u> $y = \ln x$



domain $= (0, \infty)$

$y = \ln|x|$



domain = all reals <u>except</u> $x = 0$.

<u>Check</u>: If $y = \ln|x|$ is it really true that $\frac{dy}{dx} = \frac{1}{x}$?

$$y = \ln|x| = \begin{cases} \ln x & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0 \end{cases}$$

Now find $\frac{dy}{dx}$. <u>Case 1</u>: If $x > 0$

$$\frac{dy}{dx} = \frac{d}{dx} \ln x = \frac{1}{x} \quad \checkmark$$

<u>Case 2</u>: If $x < 0$

$$\frac{dy}{dx} = \frac{d}{dx} \ln(-x) = \frac{1}{(-x)} \cdot \frac{d}{dx}[-x]$$

$$= \frac{-1}{-x} = \frac{1}{x} \quad \checkmark$$

26) $\int (x^{-2} - x^{-1} + 1 - x + x^2)\, dx$

$= -x^{-1} - \ln|x| + x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + C$

38) $\int \frac{(t+2)(t-4)}{t^2}\, dt = \int \frac{t^2-2t-8}{t^2}\, dt$

$= \int \left(\frac{t^2}{t^2} - \frac{2t}{t^2} - \frac{8}{t^2}\right) dt = \int \left(1 - \frac{2}{t} - \frac{8}{t^2}\right) dt$

$= t - 2\int \frac{1}{t}\, dt - 8 \int \frac{1}{t^2}\, dt$

$= t - 2\ln|t| - 8\int t^{-2}\, dt$

$= t - 2\ln|t| - \frac{8}{-1} t^{-1} + C$

$= t - 2\ln|t| + \frac{8}{t} + C$

47) Rate of change of total maintenance costs $= 1800e^{0.05x}$

total maintenance cost $= \int 1800\, e^{0.05x}\, dx$

where x = age (years) of the house

a) Formula for total cost $= \int 1800\, e^{0.05x}\, dx$

$M(x) = \frac{1800}{0.05} e^{0.05x} + C = 36{,}000\, e^{0.05x} + C$

Total cost is zero at $x = 0$

$0 = M(0) = 36{,}000\, e^{0.05(0)} + C = 36{,}000 + C$

so $C = -36{,}000$

47 cont'd)

$$M(x) = 36{,}000\, e^{0.05x} - 36{,}000$$
$$= 36{,}000\,(e^{0.05x} - 1)$$

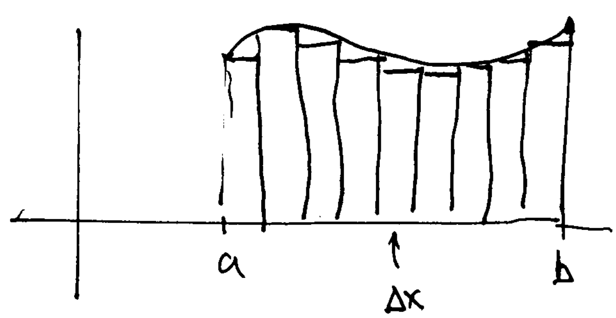
b) What are the total maintenance cost in the first 5 years?

$$M(5) = 36{,}000\, e^{0.05(5)} - 36{,}000$$
$$= 36{,}000\,(e^{0.05(5)} - 1)$$
$$= 36{,}000\,(e^{0.25} - 1)$$
$$= 10{,}225 \text{ dollars}$$

5.3 Definite Integrals and Areas

Area under a curve

$y = f(x)$



Area under curve $\approx$ Sum of areas of the rectangles

Defn: Let f be continuous on $[a, b]$. The definite integral of f from a to b is defined as

A number, not a function

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \left[f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\cdot\Delta x \right]$$

area under the curve between a and b.

area of one rectangle