Find $f'(x)$ for $f(x) = \frac{1}{x}$.

Step 0 $\quad f(x) = \frac{1}{x}$

Step 1 $\quad f(x+h) = \frac{1}{x+h}$

Step 2 $\quad f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x}$

$$= \frac{1}{x+h} \cdot \frac{x}{x} - \frac{1}{x} \cdot \frac{x+h}{x+h}$$

$$= \frac{x - (x+h)}{x(x+h)} = \frac{-h}{x(x+h)}$$

Step 3 $\quad \frac{f(x+h) - f(x)}{h} = \frac{1}{h}\left[f(x+h) - f(x)\right]$

$$= \frac{1}{h} \cdot \frac{-h}{x(x+h)} = \frac{-1}{x(x+h)}$$

Step 4 $\quad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x(x+0)}$

$$= \frac{-1}{x^2} = -x^{-2}$$

Hint for §2.2 #23) $f(x) = \sqrt{x}$ Find $f'(x)$ at $x=4$.

<u>TRICK</u>: In doing the algebra, rationalize the <u>numerator</u>.

[This page added after class ended.]

§2.2 23) Use the definition of derivative to find the slope of the tangent line of the graph of $f(x) = \sqrt{x}$ at $x = 4$.

Step 0: $f(x) = \sqrt{x}$

Step 1: $f(x+h) = \sqrt{x+h}$

Step 2: $f(x+h) - f(x) = \sqrt{x+h} - \sqrt{x}$

Step 3: $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$ ← TRICK: Rationalize the numerator, by multiplying the top and bottom by the conjugate of the numerator.

$$= \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

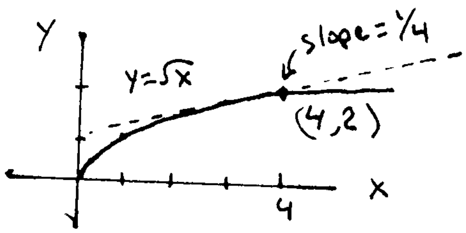
$$= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \frac{x+h - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

Step 4: $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} \dfrac{1}{\sqrt{x+h} + \sqrt{x}}$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\therefore \left\{\begin{matrix}\text{slope at} \\ x = 4\end{matrix}\right\} = f'(4) = \frac{1}{2\sqrt{4}} = \boxed{\frac{1}{4}}$$



Remark: This is the answer we're expecting: by the Power Rule, if $f(x) = x^{1/2}$ then $f'(x) = \frac{1}{2} x^{-1/2} = \dfrac{1}{2\sqrt{x}}$.

§2.3 48) $C(x)$ = cost of buying x licenses of some software (dollars)

$$C(x) = 168\, x^{5/6} \text{ dollars}$$

$$\text{so } C(1) = 168 \cdot 1^{5/6} = 168 \text{ dollars}$$

$$C(64) = 168 \cdot 64^{5/6} = 168 \cdot \sqrt[6]{64}^{\,5}$$

$$= 168 \cdot 2^5 = 168 \cdot 32$$

$$= 5376 \text{ dollars}$$

$$MC = \text{marginal cost} = C'(x)$$

a) Find and interpret $MC(1) = C'(1)$

b) " " " $MC(64) = C'(64)$.

$$C(x) = 168\, x^{5/6}$$

$$C'(x) = \frac{d}{dx}\left[168\, x^{5/6}\right] = 168 \cdot \frac{d}{dx}\left[x^{5/6}\right]$$

$$= 168 \cdot \frac{5}{6}\, x^{\frac{5}{6}-1} = 140\, x^{-1/6}$$

$$= 140 \cdot \frac{1}{\sqrt[6]{x}}$$

a) $C'(1) = 140 \cdot \frac{1}{\sqrt[6]{1}} = 140$ dollars/license

what it costs to buy the 2nd license

b) $C'(64) = 140 \cdot \frac{1}{\sqrt[6]{64}} = 140 \cdot \frac{1}{2} = 70$ dollars/license