Math 121 Mon 3 Apr 2017

Warmup/review

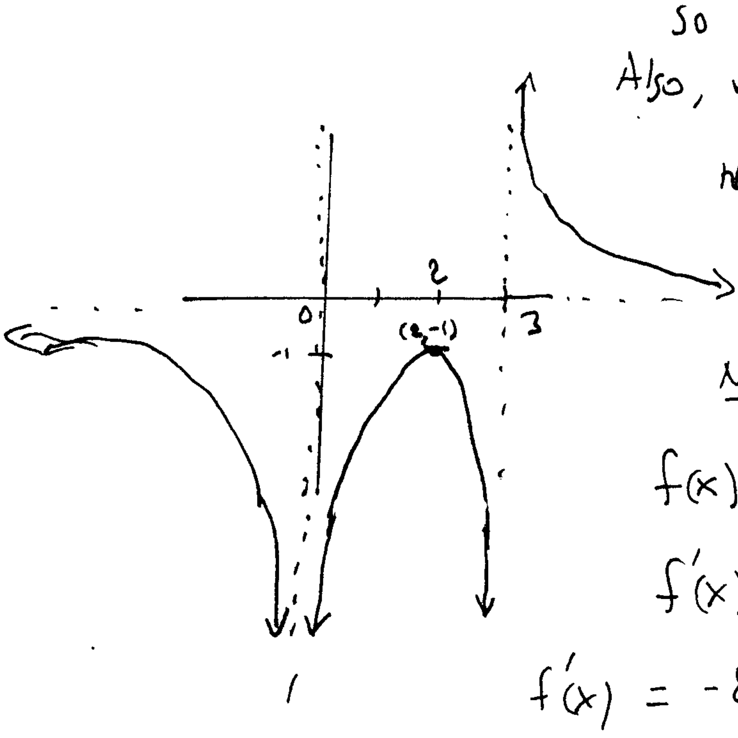
§3.1 59) $f(x) = \dfrac{4}{x^2(x-3)}$

zeros of numerator? None. So no x-intercepts.

" " denominator? 0, 0, 3

So domain is all reals except 0, 3.

Also, vertical asymptotes of $x=0$ and $x=3$



horizontal asymptote? Yes: $y=0$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{4}{x^2(x-3)} = 0$$

NO y-intercept, for 0 is not in the domain.

$$f(x) = 4x^{-2}(x-3)^{-1}$$

$$f'(x) = \frac{d}{dx}\left[4x^{-2}\right]\cdot(x-3)^{-1} + 4x^{-2}\cdot\frac{d}{dx}\left[(x-3)^{-1}\right]$$

$$f'(x) = -8x^{-3}(x-3)^{-1} + 4x^{-2}\cdot\Big[-(x-3)^{-2}\cdot\underbrace{\tfrac{d}{dx}(x-3)}_{1}\Big]$$

$$f'(x) = -8x^{-3}(x-3)^{-1} - 4x^{-2}(x-3)^{-2}$$

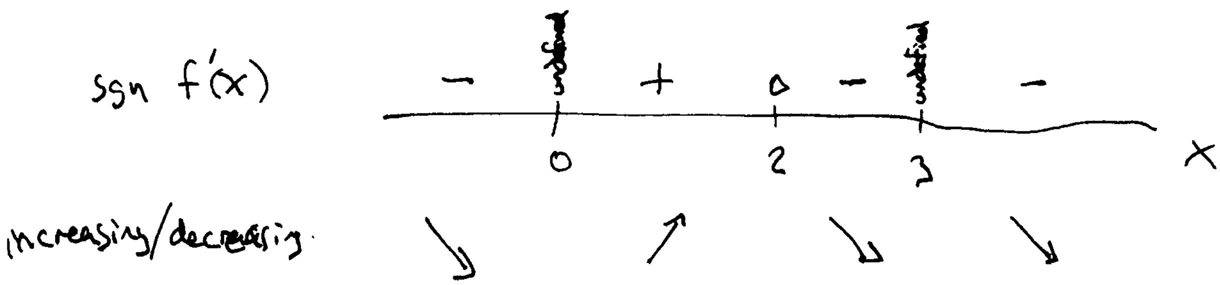
Factor the least powers of x and of (x-3):

$$f'(x) = -4x^{-3}(x-3)^{-2}\left[2(x-3) + x\right]$$

$$f'(x) = \frac{-4(2x-6+x)}{x^3(x-3)^2} = \frac{-4(3x-6)}{x^3(x-3)^2}$$

$$0 = f'(x) = \frac{-12(x-2)}{x^3(x-3)^2} \Rightarrow$$ $x=2$ is the only critical number

Sign diagram for $f'(x) = \dfrac{-12(x-2)}{x^3(x-3)^2}$



What $\underline{\text{is}}$ the relative max? We know it occurs at $x=2$

$$f(2) = \frac{4}{(2)^2(2-3)} = \frac{4}{4(-1)} = -1$$

so rel. max. at $(x,y) = (2,-1)$

Ch 3 review
p241 38)

open-topped can has volume
$$V = \pi r^2 h = 8\pi \text{ in}^3$$
Minimize the area:
$$A = 2\pi r^2 + 2\pi r h$$

Use: $h = \dfrac{8\pi}{\pi r^2}$ inches $= \dfrac{8}{r^2}$ inch

$$A(r) = 2\pi r^2 + 2\pi r \cdot \frac{8}{r^2} = 2\pi r^2 + 16\pi r^{-1}$$

$$0 = A'(r) = 4\pi r - 16\pi r^{-2} = 4\pi r - \frac{16\pi}{r^2}$$

multiply by r^2: $0 = 4\pi r^3 - 16\pi \Rightarrow 4\pi r^3 = 16\pi$

$r^3 = 4$

$\boxed{r = \sqrt[3]{4} \text{ inches}}$

Why a minimum?
$f''(\sqrt[3]{4}) > 0$

So $h = \dfrac{8}{(\sqrt[3]{4})^2} = \dfrac{8}{4^{2/3}} = \dfrac{2^3}{2^{4/3}} = 2^{5/3} = 2 \cdot 2^{2/3} = \boxed{2\sqrt[3]{4} \text{ inches}}$.

Intro to §3.6 Implicit differentiation.

2) $y^2 = x^4$ "implicitly" defines y as a function of x.
But we don't want to solve for y.
Yet, we want to find $\frac{dy}{dx}$.

$$\frac{d}{dx}[y^2] = \frac{d}{dx}[x^4]$$ Note: x is the independent variable.

chain rule!

$$2y \cdot \frac{dy}{dx} = 4x^3$$ solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{4x^3}{2y}$$

$$\boxed{\frac{dy}{dx} = \frac{2x^3}{y}}$$