5.3 Definite Integrals (cont'd)

55) $\int_1^3 (9x^2 + x^{-1})\,dx$

$= (3x^3 + \ln x)\Big|_1^3$

$= (3\cdot 3^3 + \ln 3) - (3\cdot 1^3 + \ln 1)$

$= 81 + \ln 3 - (3 + 0)$

$= 78 + \ln 3$

66) $\int_1^2 \frac{(x+1)^2}{x^2}\,dx = \int_1^2 \frac{x^2+2x+1}{x^2}\,dx$

$= \int_1^2 \left(\frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2}\right) dx$

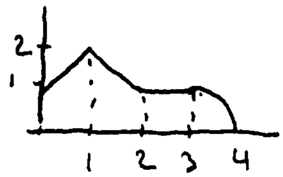
$= \int_1^2 (1 + 2\frac{1}{x} + x^{-2})\,dx$

$= (x + 2\ln|x| - x^{-1})\Big|_1^2$

$= (2 + 2\ln 2 - \frac{1}{2}) - (1 + \cancel{2\ln 1} - 1)^{\to 0}$

$= \frac{3}{2} + 2\ln 2 = \frac{3}{2} + \ln 4$

76) Here is the graph of $y=f(x)$

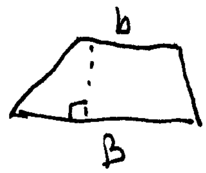


made up of line segments and a quarter circle.

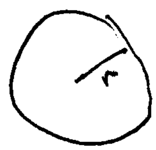
Find $\int_0^4 f(x)\,dx$.

$$= \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^4 f(x)\,dx$$

Facts:



Area of a trapezoid $= \frac{1}{2}(b+B)h$



Area of a circle $= \pi r^2$

$$\int_0^1 f(x)\,dx = \frac{1}{2}(1+2)1 = \frac{3}{2}$$

$$\int_1^2 f(x)\,dx = 3/2$$

$$\int_2^3 f(x)\,dx = 1^2 = 1$$

$$\int_3^4 f(x)\,dx = \frac{1}{4}\pi 1^2 = \frac{\pi}{4}$$

$$\int_0^4 f(x)\,dx = \text{Total area} = \frac{3}{2} + \frac{3}{2} + 1 + \frac{\pi}{4} = 4 + \frac{\pi}{4}$$

5.3 82) The marginal cost function $MC(x) = 8e^{-0.01x}$

$$\left\{\begin{array}{l}\text{Total cost}\\ \text{of producing the}\\ \text{first 100 items}\end{array}\right\} = \int_0^{100} MC(x)\,dx$$

$$= \int_0^{100} 8e^{-0.01x}\,dx = 8 \cdot \frac{1}{-0.01} e^{-0.01x} \Big|_0^{100}$$

$$= -800\, e^{-0.01x} \Big|_0^{100}$$

$$= -800\, e^{-0.01(100)} - \left(-800 e^{-0.01(0)}\right)$$

$$= -800\, e^{-1} + 800$$

$$= -800\,(0.3679) + 800$$

$$= 800\,(1 - 0.3679) = 800\,(0.6321)$$

$$= 505.70 \text{ dollars}$$