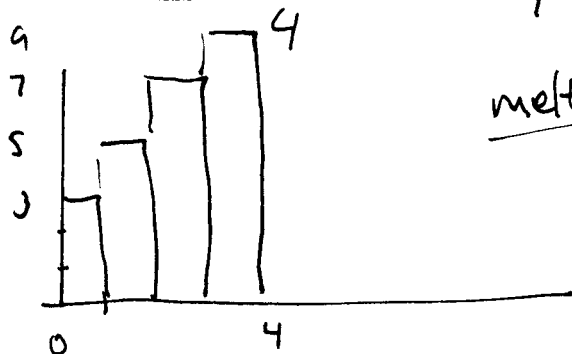


5.4 Average value and Area between curves

Average value

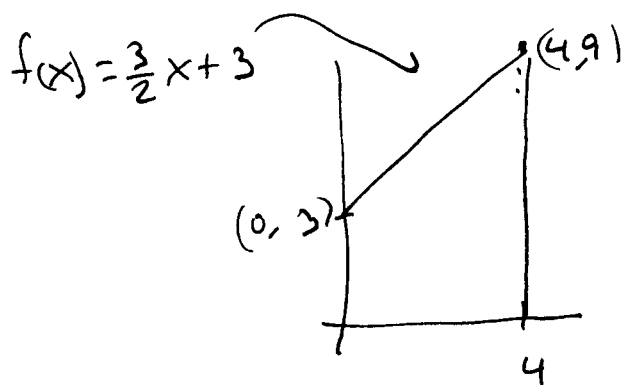
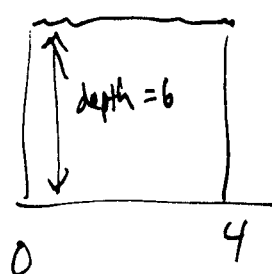
ex.: what is the average of 3, 5, 7 and 9.

$$\frac{3+5+7+9}{4} = \frac{24}{4} = 6$$

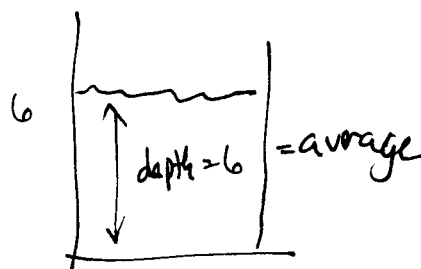


↑
ice cubes
in an aquarium

melts



melts



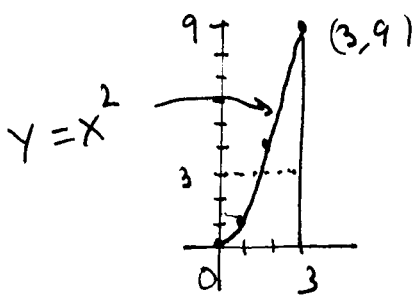
Defn: Average of a function $f(x)$ between $x=a$ and $x=b$
is $\frac{1}{b-a} \int_a^b f(x) dx$

(2)

ex. Find the average value of $f(x) = \frac{3}{2}x + 3$ on the interval $[0, 4]$.

$$\begin{aligned} & \frac{1}{4-0} \int_0^4 \left(\frac{3}{2}x + 3 \right) dx \\ &= \frac{1}{4} \int_0^4 \left(\frac{3}{2}x + 3 \right) dx = \frac{1}{4} \left(\frac{3}{4}x^2 + 3x \right) \Big|_0^4 \\ &= \frac{1}{4} \left(\frac{3}{4} \cdot 4^2 + 3 \cdot 4 \right) - \frac{1}{4} \left(\frac{3}{4} \cdot 0^2 + 3 \cdot 0 \right) \\ &= \frac{1}{4} (12 + 12) = \frac{1}{4} (24) = 6 \end{aligned}$$

ex: Find the avg value value of $f(x) = x^2$ on the interval $[0, 3]$.



$$\begin{aligned} \{ \text{avg value} \} &= \frac{1}{3-0} \int_0^3 x^2 dx \\ &= \frac{1}{3} \left(\frac{1}{3} x^3 \right) \Big|_0^3 = \frac{1}{9} \cdot 3^3 - \frac{1}{9} \cdot 0^3 \\ &= \frac{1}{9} \cdot 27 - 0 \\ &= 3 \end{aligned}$$

16) Find the avg value of $f(x) = e^{-2x}$ on $[0, 1]$

$$\left\{ \begin{array}{l} \text{avg} \\ \text{value} \end{array} \right\} = \frac{1}{1-0} \int_0^1 e^{-2x} dx$$

$$= \int_0^1 e^{-2x} dx = \frac{1}{-2} e^{-2x} \Big|_0^1$$

$$= -\frac{1}{2} e^{-2 \cdot 1} + \frac{1}{2} e^{-2 \cdot 0}$$

$$= -\frac{1}{2} e^{-2} + \frac{1}{2} \approx \frac{1}{2} (1 - e^{-2})$$

$$= \frac{1}{2} (1 - 0.135335)$$

$$= \frac{1}{2} (0.86466) = 0.43233$$