Basic Integration Formulas

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C \qquad (n \neq -1)$
2. $\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \int \frac{dx}{x} = \ln|x| + C$
3. $\int e^x\,dx = e^x + C$
4. $\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$

Substitution Formulas

5. $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C \qquad (n \neq -1)$
6. $\int u^{-1}\,du = \int \frac{1}{u}\,du = \int \frac{du}{u} = \ln|u| + C$
7. $\int e^u\,du = e^u + C$

Integration by Parts Formula

8. $\int u\,dv = uv - \int v\,du$

A Short Table of Integrals

Forms Involving $ax + b$:

9. $\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + C$
10. $\int \frac{1}{(ax+b)(cx+d)}\,dx = \frac{1}{ad-bc}\ln\left|\frac{ax+b}{cx+d}\right| + C$
11. $\int \frac{x}{(ax+b)(cx+d)}\,dx = \frac{1}{ad-bc}\left(\frac{d}{c}\ln|cx+d| - \frac{b}{a}\ln|ax+b|\right) + C$
12. $\int \frac{1}{x^2(ax+b)}\,dx = -\frac{1}{b}\left(\frac{1}{x} + \frac{a}{b}\ln\left|\frac{x}{ax+b}\right|\right) + C$

Forms Involving $\sqrt{ax+b}$:

13. $\int \frac{x}{\sqrt{ax+b}}\,dx = \frac{2ax-4b}{3a^2}\sqrt{ax+b} + C$
14. $\int \frac{1}{x\sqrt{ax+b}}\,dx = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C \qquad (b > 0)$

Forms Involving $x^2 - a^2$ and $a^2 - x^2$:

15. $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$
16. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$

Forms Involving $\sqrt{x^2 \pm a^2}$:

17. $\int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$
18. $\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

Forms Involving $\sqrt{a^2 \pm x^2}$:

19. $\int \frac{\sqrt{a^2 \pm x^2}}{x}\,dx = \sqrt{a^2 \pm x^2} - a\ln\left|\frac{a + \sqrt{a^2 \pm x^2}}{x}\right| + C$
20. $\int \frac{1}{x\sqrt{a^2 \pm x^2}}\,dx = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 \pm x^2}}{x}\right| + C$

Forms Involving e^{ax} and $\ln x$:

21. $\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$
22. $\int (\ln x)^n\,dx = x(\ln x)^n - n\int (\ln x)^{n-1}\,dx$
23. $\int x^n \ln x\,dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + C \qquad (n \neq -1)$

Trigonometric Formulas

See also page 616.

$\int \sin x\,dx = -\cos x + C$ $\quad \int \sec^2 x\,dx = \tan x + C$ $\quad \int \sec x \tan x\,dx = \sec x + C$

$\int \cos x\,dx = \sin x + C$ $\quad \int \csc^2 x\,dx = -\cot x + C$ $\quad \int \csc x \cot x\,dx = -\csc x + C$