Math 122 - Fall 2015

Thu 27 Aug

5.6 Integration by Substitution

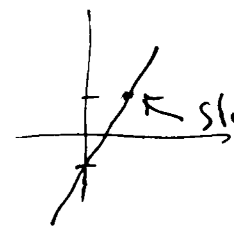
= Chain Rule + Antiderivatives

Notation: Derivative and Differential

ex $\frac{d}{dx}[x^3] = 3x^2$ derivative

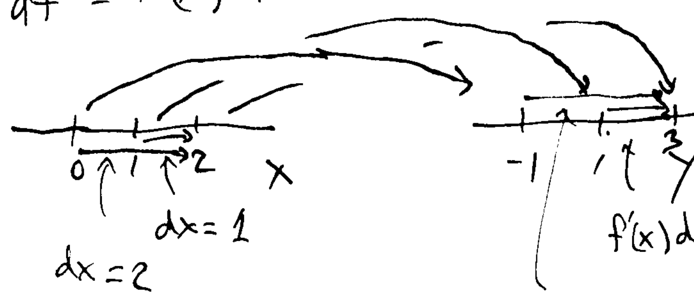
$d(x^3) = 3x^2\,dx$ differential

ex: $y = f(x) = 2x - 1$



$\frac{df}{dx} = f'(x) = 2$

$df = f'(x)\,dx = 2dx$



$f'(x)dx = 2dx = 2(1) = 2$

$f'(x)\,dx = 2dx = 2(2) = 4$

ex: If $f(x) = \ln x$, then $df = \frac{1}{x}dx$

If $f(x) = e^{3x}$ then $df = f'(x)dx$
$= 3e^{3x}dx$

Reason: $\frac{d}{dx}e^{3x} = e^{3x}\cdot\frac{d(3x)}{dx}$
$= 3e^{3x}$

If $f(x) = x^2 + 8x + 15$
then $df = (2x+8)dx$.

Integration by Substitution

Remark: Do not leave out the dx in an integral.

ex: $\int \left(x^5 + e^{3x} + \frac{1}{x}\right) dx$ ← we need the dx.

$= \frac{1}{6}x^6 + \frac{1}{3}e^{3x} + \ln|x| + C$

ex: $\int 2x\sqrt{x^2+1}\,dx$ Must use substitution.

Rules of thumb for choosing u:

1. Let u = (whatever is in parentheses)
2. If the integrand contains a rational function
   u = denominator
3. If the integrand contains $e^{\text{expression}}$
   Let u = expression.

example (cont'd)

$u = x^2+1$
$du = 2x\,dx$

$\int 2x(x^2+1)^{1/2}\,dx$

$= \int (x^2+1)^{1/2} \cdot 2x\,dx$

$= \int u^{1/2}\,du$

$= \frac{2}{3}u^{3/2} + C = \frac{2}{3}(x^2+1)^{3/2} + C$

<u>Check</u> our antiderivative by taking the derivative

$$\frac{d}{dx}\left[\frac{2}{3}(x^2+1)^{3/2}+C\right]$$

$$= \frac{2}{3}\frac{d}{dx}\left[(x^2+1)^{3/2}\right] + \frac{d}{dx}[C] \to 0$$

$$= \frac{2}{3}\cdot\frac{3}{2}(x^2+1)^{1/2}\cdot\frac{d}{dx}[x^2+1]$$

$$= (x^2+1)^{1/2}\cdot 2x = 2x\sqrt{x^2+1} \quad \checkmark$$

<u>ex</u>: $\int (12x+30)e^{x^2+5x}\,dx$

$$\boxed{\begin{aligned} u &= x^2+5x \\ du &= (2x+5)dx \end{aligned}}$$

$$= 6\int (2x+5)e^{x^2+5x}\,dx$$

$$= 6\int e^u\,du$$

$$= 6e^u + C$$

$$= 6e^{x^2+5x} + C$$