Warmup

(1) $$\int \frac{2x}{x^2-1}\,dx = \int \frac{du}{u} = \ln|u| + C$$
$$= \ln|x^2-1| + C$$

$u = x^2-1$
$du = 2x\,dx$

(2) $$\int \sqrt{3x+5}\,dx = \int (3x+5)^{1/2}\,dx$$
$$= \frac{1}{3}\cdot\frac{2}{3}(3x+5)^{3/2} + C$$
$$= \frac{2}{9}(3x+5)^{3/2} + C$$

(3) $$\int \frac{1}{2x+1}\,dx = \frac{1}{2}\ln|2x+1| + C$$

(4) $$\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$$

(5) $$\int 5xe^{3x}\,dx = \underbrace{(5x)}_{u}\underbrace{\left(\tfrac{1}{3}e^{3x}\right)}_{v} - \int \underbrace{(5)}_{du}\underbrace{\left(\tfrac{1}{3}e^{3x}\right)}_{v}dx$$

By parts:

To be differentiated	To be integrated
$u = 5x$	$dv = e^{3x}\,dx$
$du = 5\,dx$	$v = \frac{1}{3}e^{3x}$

Using:
$$\int u\,dv = uv - \int v\,du$$

$$= \frac{5}{3}xe^{3x} - \frac{5}{3}\int e^{3x}\,dx$$

$$= \frac{5}{3}xe^{3x} - \frac{5}{3}\cdot\frac{1}{3}e^{3x} + C$$

$$= \frac{5}{3}xe^{3x} - \frac{5}{9}e^{3x} + C$$

(6) $$\int x^2\ln x\,dx = (\ln x)\left(\frac{x^3}{3}\right) - \int\left(\frac{1}{x}\right)\left(\frac{x^3}{3}\right)dx$$

Hint:

Diff	Int
$u = \ln x$	$dv = x^2\,dx$
$du = \frac{1}{x}\,dx$	$v = \frac{x^3}{3}$

$$= \frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\,dx$$

$$= \frac{1}{3}x^3\ln x - \frac{1}{3}\cdot\frac{x^3}{3} + C$$

$$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$$

Tabular integration by parts

ex: $\int (2x+1)\, e^{5x}\, dx = (2x+1)\left(\frac{1}{5}e^{5x}\right) - \int (2)\left(\frac{1}{5}e^{5x}\right) dx$

Standard way: of parts

$u = 2x+1 \qquad dv = e^{5x}\,dx$

$du = 2\,dx \qquad v = \frac{1}{5}e^{5x}$

use: $\int u\,dv = uv - \int v\,du$

$= \frac{1}{5}(2x+1)e^{5x} - \frac{2}{5}\int e^{5x}\,dx$

$= \frac{1}{5}(2x+1)e^{5x} - \frac{2}{5}\cdot\frac{1}{5}e^{5x} + C$

$= \frac{1}{5}(2x+1)e^{5x} - \frac{2}{25}e^{5x} + C$

same problem by
Tabular integration by parts

signs	differentiate	integrate
		e^{5x}
$+$	$2x+1$	$\frac{1}{5}e^{5x}$
$-$	2	$\frac{1}{25}e^{5x}$
$+$	0	

$\int (2x+1)\, e^{5x}\, dx = \text{Answer} = (2x+1)\left(\frac{1}{5}e^{5x}\right) - (2)\left(\frac{1}{25}e^{5x}\right) + C$

$= \frac{1}{5}(2x+1)e^{5x} - \frac{2}{25}e^{5x} + C$

ex: $\int x^2 e^{7x}\,dx = (x^2)\left(\frac{1}{7}e^{7x}\right) - \int (2x)\left(\frac{1}{7}e^{7x}\right)dx$

one Tabular integration by parts:

$$= (x^2)\left(\frac{1}{7}e^{7x}\right) - (2x)\left(\frac{1}{49}e^{7x}\right) + \int (2)\left(\frac{1}{49}e^{7x}\right)dx$$

signs	diff	int
$+$	x^2	e^{7x}
$-$	$2x$	$\frac{1}{7}e^{7x}$
$+$	2	$\frac{1}{49}e^{7x}$
$-$	0	$\frac{1}{343}e^{7x}$

$$= (x^2)\left(\frac{1}{7}e^{7x}\right) - (2x)\left(\frac{1}{49}e^{7x}\right) + (2)\left(\frac{1}{343}e^{7x}\right) + C$$