Quiz 1 #4 by "Tabular integration by parts"

$$\int (x+10)^2 e^{5x}\,dx = (x+10)^2\left(\tfrac{1}{5}e^{5x}\right) - 2(x+10)\left(\tfrac{1}{25}e^{5x}\right) + (2)\left(\tfrac{1}{125}e^{5x}\right) + C$$

signs		diff	int
$+$	$\longrightarrow$	$(x+10)^2$	e^{5x}
$-$	$\longrightarrow$	$2(x+10)$	$\frac{1}{5}e^{5x}$
$+$	$\longrightarrow$	2	$\frac{1}{25}e^{5x}$
		0	$\frac{1}{125}e^{5x}$

$$= \frac{1}{5}(x+10)^2 e^{5x} - \frac{2}{25}(x+10)e^{5x} + \frac{2}{125}e^{5x} + C$$

#2) $\int x^4 \ln x\,dx = (\ln x)\left(\frac{x^5}{5}\right) - \int \left(\frac{1}{x}\right)\left(\frac{x^5}{5}\right)dx$

signs		diff		int
$+$	$\longrightarrow$	$\ln x$		x^4
$-$	$\longrightarrow$	$\frac{1}{x}$	$\longrightarrow$	$\frac{x^5}{5}$

$$= \frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^4\,dx$$

$$= \frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + C$$

Note: ① In following a diagonal arrow, write the product without an integral sign.
② In following all-horizontal arrows, write the product with an integral sign.

example (from text): $S(x) = \int \frac{x}{\sqrt{x+9}}\,dx = (*)$

Use formula #13: $\int \frac{x}{\sqrt{ax+b}}\,dx = \frac{2ax-4b}{3a^2}\sqrt{ax+b} + C$

[pattern recognition]

with $a=1$, $b=9$:

$$(*) = \frac{2x-4(9)}{3\cdot 1^2}\sqrt{x+9} + C$$

$$= \frac{2x-36}{3}\sqrt{x+9} + C$$

20) $\int x^{99} \ln x \, dx = (*)$

Use #23 $\int x^n \ln x \, dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C$

with $n = 99$.

$$(*) = \frac{1}{100} x^{100} \ln x - \frac{1}{100^2} x^{100} + C$$

$$= \frac{1}{100} x^{100} \ln x - \frac{1}{10000} x^{100} + C$$

22) $\int (\ln x)^2 \, dx = (*)$

Use #22 $\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1} dx$

first with $n=2$:

$$(*) = x(\ln x)^2 - 2\int (\ln x)^1 dx = (**)$$

now use #22 with $n=1$:

$$(**) = x(\ln x)^2 - 2\left[x(\ln x)^1 - 1\int (\ln x)^0 dx \right]$$

$$= x(\ln x)^2 - 2x\ln x + 2\int 1 \, dx$$

$$= x(\ln x)^2 - 2x\ln x + 2x + C$$

32) $$\int \frac{e^{2t}}{1-e^t}\,dt = (*)$$

Aw shoot!! This doesn't <u>yet</u> match any formula.

Let's try substitution.

$$\boxed{\begin{aligned} \text{Try} \quad u &= e^t \\ du &= e^t\,dt \end{aligned}}$$

$$(*) = \int \frac{e^t \cdot e^t\,dt}{1-e^t} = \int \frac{u\,du}{1-u}$$

$$\boxed{\begin{array}{l} \text{Use \#9} \\ \text{with } u = x \\ \text{and } a = -1 \\ \text{and } b = 1 \end{array}}$$

$$= \int \frac{u}{-u+1}\,du$$

$$= \frac{u}{a} - \frac{b}{a^2}\ln|au+b| + C$$

$$= \frac{u}{-1} - \frac{1}{(-1)^2}\ln|-u+1| + C$$

$$= -e^t - \ln|-e^t+1| + C$$

$$= -e^t - \ln|1-e^t| + C$$

34) $\int x^2 \sqrt{x^6+1}\, dx$

Aw shoot... doesn't match...

Note: $x^6 = (x^3)^2$

Let

$u = x^3$

$du = 3x^2 dx$

$$= \frac{1}{3} \int \left[(x^3)^2 + 1 \right]^{1/2} 3x^2 dx$$

$$= \frac{1}{3} \int \sqrt{u^2+1}\, du$$

This matches #17 with $u = x$ and $a = 1$ and "$\pm$" $= +$

[The rest was completed after class ended.]

$$= \frac{1}{3} \left[\frac{u}{2} \sqrt{u^2+1} + \frac{1}{2} \ln \left| u + \sqrt{u^2+1} \right| \right] + C$$

$$= \frac{1}{3} \left[\frac{x^3}{2} \sqrt{x^6+1} + \frac{1}{2} \ln \left| x^3 + \sqrt{x^6+1} \right| \right] + C$$

$$= \frac{1}{6} x^3 \sqrt{x^6+1} + \frac{1}{6} \ln \left| x^3 + \sqrt{x^6+1} \right| + C$$