Math 122 Tue 15 Sep 2015

6.3 Improper Integrals

Remark: Don't tell anyone, but you sometimes get the right answer by "abusing notation" by treating ∞ as if it's just a really big number.

abuse of notation

ex (#19) $$\int_2^{\infty} 3x^{-4}\,dx = \frac{3x^{-3}}{-3}\Big|_2^{\infty}$$

$$= -x^{-3}\Big|_2^{\infty} = \frac{-1}{x^3}\Big|_2^{\infty} = \frac{-1}{\infty^3} + \frac{1}{2^3}$$

$$= -0 + \frac{1}{8} = \boxed{\frac{1}{8}}$$

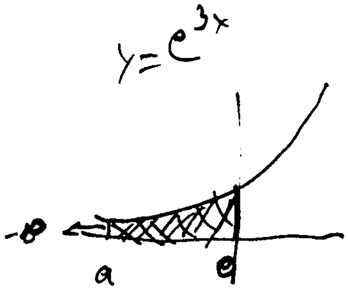
23) $$\int_1^{\infty} \frac{1}{x^{1.01}}\,dx = \int_1^{\infty} x^{-1.01}\,dx = \frac{x^{-0.01}}{-0.01}\Big|_1^{\infty}$$

$$= -\frac{100}{x^{.01}}\Big|_1^{\infty} = -\frac{100}{\sqrt[100]{x}}\Big|_1^{\infty}$$

$$= -\frac{100}{\sqrt[100]{\infty}} + \frac{100}{\sqrt[100]{1}} = -0 + 100$$

$$= \boxed{100}$$

33) $\displaystyle\int_{-\infty}^{0} e^{3x}\,dx = \lim_{a\to-\infty} \int_a^0 e^{3x}\,dx$



Step 1 $\displaystyle\int_a^0 e^{3x}\,dx = \frac{1}{3}e^{3x}\Big|_a^0$

$$= \frac{1}{3}e^0 - \frac{1}{3}e^{3a}$$

$$= \frac{1}{3} - \frac{1}{3}e^{3a}$$

Step 2 $\displaystyle\lim_{a\to-\infty} \int_a^0 e^{3x}\,dx$

$$= \lim_{a\to-\infty}\left[\frac{1}{3} - \frac{1}{3}e^{3a}\right]$$

$$= \frac{1}{3} - 0 = \frac{1}{3}$$

39) $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{1+e^x}\,dx$

Hmm. Step 0 $\displaystyle\int \frac{e^x}{1+e^x}\,dx = \int \frac{du}{u} = \ln|u| + C$

$$= \ln|1+e^x| + C$$

$u = 1+e^x$
$du = e^x\,dx$

$$\int_{-\infty}^{\infty} \frac{e^x}{1+e^x}\,dx = \lim_{a\to-\infty} \int_a^0 \frac{e^x}{1+e^x}\,dx + \lim_{b\to\infty} \int_0^b \frac{e^x}{1+e^x}\,dx$$

$\uparrow$ 1 $\qquad\qquad$ $\uparrow$ 2

1 Step 1

$$\int_a^0 \frac{e^x}{1+e^x}\,dx = \ln|1+e^x|\Big|_a^0$$

$$= \ln|1+e^0| - \ln|1+e^a|$$

$$= \ln 2 - \ln|1+e^a|$$

Step 2

$$\lim_{a\to-\infty}\left(\ln 2 - \ln|1+e^a|\right)$$

$$= \ln 2 - \ln|1+0| = \ln 2 - \ln 1$$

$$= \boxed{\ln 2}$$

2 Step 1

$$\int_0^b \frac{e^x}{1+e^x}\,dx = \ln|1+e^x|\Big|_0^b$$

$$= \ln|1+e^b| - \ln|1+e^0|$$

$$= \ln|1+e^b| - \ln 2$$

Step 2

$$\lim_{b\to\infty}\left(\ln|1+e^b| - \ln 2\right) = \infty$$

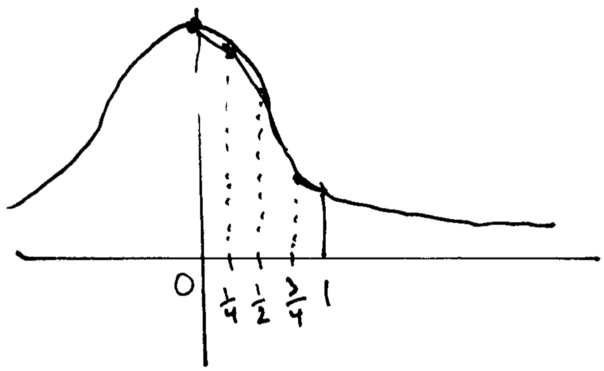
That is, this limit does not exist.

OVERALL ANSWER: $\int_{-\infty}^{\infty} \frac{e^x}{1+e^x}\,dx$ diverges.

6.4 Numerical Integration

8) Use the Trapezoid Rule with $n=4$ to approximate $\int_0^1 e^{-x^2}\,dx$

[Graphing calculator answer $= .7468241$]



Idea: Area of a trapezoid $= \frac{1}{2}(b+B)h$

Area of 1st trapezoid

$$= \frac{1}{2}\left[f(0) + f\left(\tfrac{1}{4}\right)\right]\Delta x$$

NOTE: $\Delta x = \frac{b-a}{n}$

$$= \frac{1-0}{4} = \frac{1}{4}$$

Sum of area of 4 trapezoids $= \frac{1}{2}\left[f(0) + f\left(\tfrac{1}{4}\right)\right]\Delta x + \frac{1}{2}\left[f\left(\tfrac{1}{4}\right) + f\left(\tfrac{1}{2}\right)\right]\Delta x$

$$+ \frac{1}{2}\left[f\left(\tfrac{1}{2}\right) + f\left(\tfrac{3}{4}\right)\right]\Delta x + \frac{1}{2}\left[f\left(\tfrac{3}{4}\right) + f(1)\right]\Delta x$$

$$= \left[\frac{1}{2}f(0) + f\left(\tfrac{1}{4}\right) + f\left(\tfrac{1}{2}\right) + f\left(\tfrac{3}{4}\right) + \frac{1}{2}f(1)\right]\Delta x$$

$1/4$

NOTE: The weights are $\frac{1}{2}, 1, 1, 1, \frac{1}{2}$

\# 8 cont'd)

x	$f(x)=e^{-x^2}$	weights	$f(x)\cdot wts$
0	1	$\frac{1}{2}$	.5
.25	.93941	1	.93941
.5	.7788	1	.7788
.75	.56978	1	.56978
1	.36788	$1/2$	.18394

$2.971936 = \sum f(x)\cdot wt$

$$\left[\sum f(x)\cdot wt\right]\cdot \Delta x = (2.971936)\left(\tfrac{1}{4}\right)$$
$$= \boxed{.742\,984\,0978}$$

Compare to .74682413 ← better