Math 122 Thu 17 Sep 2015

Warmup

(1) $$\int_{-\infty}^{-1} \frac{1}{\sqrt[3]{x^4}}\,dx = \int_{-\infty}^{-1} x^{-4/3}\,dx$$

$$= \lim_{a\to-\infty} \int_a^{-1} x^{-4/3}\,dx = \lim_{a\to-\infty} -3x^{-1/3}\Big|_a^{-1}$$

$$= \lim_{a\to-\infty} \left[-3(-1)^{-1/3} + 3a^{-1/3}\right]$$

$$= \lim_{a\to-\infty} \left[-3\cdot \frac{1}{\sqrt[3]{-1}} + 3\cdot\frac{1}{\sqrt[3]{a}}\right]$$

$$= \lim_{a\to-\infty} \left[3 + \frac{3}{\sqrt[3]{a}}\right] = 3+0 = \boxed{3}$$

(2) 6.3 32) $$\int_e^\infty (\ln x)^{-2}\,\frac{1}{x}\,dx = \lim_{b\to\infty}\int_e^b (\ln x)^{-2}\,\frac{1}{x}\,dx$$

Step 1 $$\int_{x=e}^{x=b} (\ln x)^{-2}\,\frac{1}{x}\,dx = \int_{u=1}^{u=\ln b} u^{-2}\,du$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

when $x=e$, $u = \ln e = 1$

when $x=b$, $u = \ln b$

$$= -u^{-1}\Big|_1^{\ln b}$$

$$= -(\ln b)^{-1} - (-1^{-1})$$

$$= -\frac{1}{\ln b} + 1$$

2 cont'd

Step 2 $\displaystyle \lim_{b \to \infty} \frac{-1}{\ln b} + 1 = -0 + 1 = \boxed{1}$

Warmup

3 6.2 #24)

$$\int \frac{1}{x(x-3)}\,dx = \int \frac{1}{(1x+0)(1x+-3)}\,dx$$

So use #10
with $a=1$
$b=0$
$c=1$
$d=-3$

$$= \int \frac{1}{(ax+b)(cx+d)}\,dx$$

$$= \frac{1}{ad-bc} \ln \left| \frac{ax+b}{cx+d} \right| + C$$

$$= \frac{1}{(1)(-3)-(0)(1)} \ln \left| \frac{1x+0}{1x+(-3)} \right| + C$$

$$= \boxed{\frac{1}{-3} \ln \left| \frac{x}{x-3} \right| + C}$$

6.4 Numerical integration – Simpson's Rule

25) Approximate $\int_0^1 e^{-x^2}\,dx$ with $n=4$.

x	$f(x)$	wts	wts·f(x)
0	1	1	1
.25	.93941	4	3.7577
.5	.7788	2	1.5576
.75	.56978	4	2.2791
1	.36788	1	0.36788

$\sum wt \cdot f(x) = 8.96226$

$$\int_0^1 e^{-x^2}\,dx \approx \frac{\Delta x}{3}\left[\sum wt \cdot f(x)\right] = .746\,855\,3798$$

$$= \frac{1}{12}\left[8.96226\right]$$

Actual: .746 824 13

because $\frac{\Delta x}{3} = \frac{1}{3}\cdot\frac{1}{4} = \frac{1}{12}$