

Random Review problems

ex: $\int (\ln x) dx = \int (\ln x) 1 dx$

$\begin{array}{ll} u = \ln x & dv = 1 dx \\ du = \frac{1}{x} dx & v = x \end{array}$	$\begin{aligned} &= (\ln x)(x) - \int (x)\left(\frac{1}{x}\right) dx \\ &= x \ln x - \int 1 dx \\ &= x \ln x - x + C \end{aligned}$
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Q1 #1: [By Tabular IBP]

<table border="0"> <thead> <tr> <th><u>signs</u></th> <th><u>diff</u></th> <th><u>int</u></th> </tr> </thead> <tbody> <tr> <td>+</td> <td>$\rightarrow x$</td> <td>e^{3x}</td> </tr> <tr> <td>-</td> <td>$\rightarrow 1$</td> <td>$\searrow \frac{1}{3} e^{3x}$</td> </tr> <tr> <td>+</td> <td>$\rightarrow 0$</td> <td>$\searrow \frac{1}{9} e^{3x}$</td> </tr> </tbody> </table>	<u>signs</u>	<u>diff</u>	<u>int</u>	+	$\rightarrow x$	e^{3x}	-	$\rightarrow 1$	$\searrow \frac{1}{3} e^{3x}$	+	$\rightarrow 0$	$\searrow \frac{1}{9} e^{3x}$	$\begin{aligned} \int x e^{3x} dx &= (x) \left(\frac{1}{3} e^{3x} \right) - (1) \left(\frac{1}{9} e^{3x} \right) + C \\ &= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C \end{aligned}$
<u>signs</u>	<u>diff</u>	<u>int</u>											
+	$\rightarrow x$	e^{3x}											
-	$\rightarrow 1$	$\searrow \frac{1}{3} e^{3x}$											
+	$\rightarrow 0$	$\searrow \frac{1}{9} e^{3x}$											

Recall: $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$

(2)

ex: $\int x^3 e^{5x} dx = (x^3) \left(\frac{1}{5} e^{5x} \right) - (3x^2) \left(\frac{1}{25} e^{5x} \right)$

$$+ (6x) \left(\frac{1}{125} e^{5x} \right) - (6) \left(\frac{1}{625} e^{5x} \right) + C$$

$$= \frac{1}{5} x^3 e^{5x} - \frac{3}{25} x^2 e^{5x} + \frac{6}{125} x e^{5x} - \frac{6}{625} e^{5x} + C$$

<u>signs</u>	<u>diff</u>	<u>int</u>
+	x^3	e^{5x}
-	$3x^2$	$\frac{1}{5} e^{5x}$
+	$6x$	$\frac{1}{25} e^{5x}$
-	6	$\frac{1}{125} e^{5x}$
	0	$\frac{1}{625} e^{5x}$

ex: $\int x^2 (x+1)^{-1/2} dx = 2x^2 (x+1)^{+1/2} - \frac{8}{3} x (x+1)^{3/2}$

$$+ \frac{16}{15} (x+1)^{5/2} + C$$

<u>signs</u>	<u>diff</u>	<u>int</u>
+	x^2	$(x+1)^{-1/2}$
-	$2x$	$2(x+1)^{1/2}$
+	2	$2 \cdot \frac{2}{3} (x+1)^{3/2}$
	0	$2 \cdot \frac{2}{3} \cdot \frac{2}{5} (x+1)^{5/2}$

p460

30)

use tables

$$\int \frac{x}{\sqrt{x+1}} dx = (*)$$

$$\text{Use \#13: } \int \frac{x}{\sqrt{ax+b}} dx = \frac{2ax-4b}{3a^2} \sqrt{ax+b} + C$$

$$\text{with } a=1 \\ b=1$$

$$\begin{aligned} (*) &= \frac{2 \cdot 1 \cdot x - 4 \cdot 1}{3 \cdot 1^2} \sqrt{x+1} + C \\ &= \frac{2x-4}{3} \sqrt{x+1} + C \end{aligned}$$

$$\text{(Sneaky) ex: } \int \frac{x+1}{(x+2)(x+3)} dx = \int \frac{x}{(x+2)(x+3)} dx + \int \frac{1}{(x+2)(x+3)} dx$$

use \#10

use \#11

etc.
[you finish]

$$\text{p460 34) } \int \frac{e^{2t}}{e^t+2} dt = \int \frac{e^t \cdot e^t dt}{e^t+2}$$

$$\begin{aligned} \text{Try } u &= e^t \\ du &= e^t dt \\ e^t + 2 &= u + 2 \\ e^{2t} &= (e^t)^2 = e^t \cdot e^t \end{aligned}$$

with

$$\begin{aligned} a &= 1 \\ b &= 2 \end{aligned}$$

x=u

$$= \int \frac{u}{u+2} du = \int \frac{u}{u+2} du = (*)$$

$$\text{Use \#9 } \int \frac{x}{ax+b} dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b| + C$$

$$(*) = \frac{u}{1} - \frac{2}{1^2} \ln|u+2| + C$$

$$= e^t - 2 \ln|e^t+2| + C$$

(4)

p460 48) $\int_{100}^{\infty} e^{-t/100} dt = \lim_{b \rightarrow \infty} \int_{100}^b e^{-t/100} dt$

$$\begin{aligned} \int_{100}^b e^{-t/100} dt &= -\frac{100}{1} e^{-t/100} \Big|_{100}^b \\ &= -100 e^{-b/100} + 100 e^{-100/100} \\ &= -100 e^{-\frac{b}{100}} + 100 e^{-1} \end{aligned}$$

$$\lim_{b \rightarrow \infty} \left[-100 e^{-\frac{b}{100}} + \frac{100}{e} \right] = 0 + \frac{100}{e} = \boxed{\frac{100}{e}}$$

p460 42) $\int_1^{\infty} \frac{1}{\sqrt[6]{x}} dx = \lim_{b \rightarrow \infty} \int_1^b x^{-1/6} dx$

$$= \lim_{b \rightarrow \infty} \left. \frac{6}{5} x^{5/6} \right|_1^b = \lim_{b \rightarrow \infty} \frac{6}{5} b^{5/6} - \frac{6}{5} \cdot 1^{5/6}$$

$$= \lim_{b \rightarrow \infty} \frac{6}{5} \sqrt[6]{b^5} - \frac{6}{5} = \infty - \frac{6}{5} = \infty$$

DIVERGENT.

7.1 Functions of two or three variables

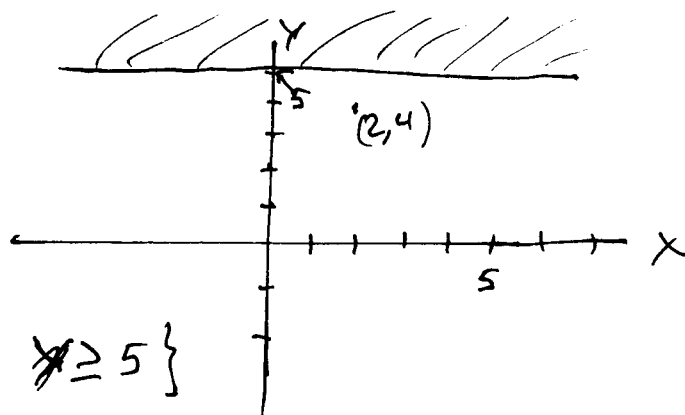
ex: $f(x, y) = x^2 + \sqrt{y-5}$

$$f(3, 9) = 3^2 + \sqrt{9-5} = 9 + \sqrt{4} = 9 + 2 = 11$$

$$f(-2, 6) = (-2)^2 + \sqrt{6-5} = 4 + \sqrt{1} = 5$$

$$f(2, 4) = 2^2 + \sqrt{4-5} = 4 + \sqrt{-1} \leftarrow \text{not real}$$

Domain of f ?



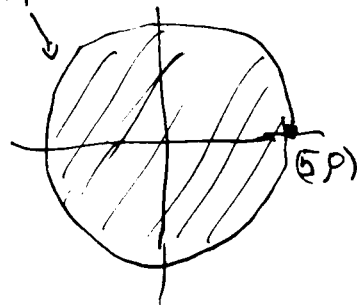
$$\text{Domain} = \{(x, y) \mid y \geq 5\}$$

ex: $f(x, y) = \sqrt{25 - x^2 - y^2}$

$$f(1, 2) = \sqrt{25 - 1^2 - 2^2} = \sqrt{25 - 5} = \sqrt{20}$$

we need $25 - x^2 - y^2 \geq 0$
 $25 \geq x^2 + y^2$

$$x^2 + y^2 = 25$$



$$\text{Domain} = \{(x, y) \mid x^2 + y^2 \leq 25\}$$

= disk of radius 5