7.1 Functions of Two (or Three) variables

10) $f(x,y) = \sqrt{75 - x^2 - y^2}$. Find $f(5,-1)$.

$$f(5,-1) = \sqrt{75 - 5^2 - (-1)^2} = \sqrt{75 - 25 - 1}$$

$$= \sqrt{49} = 7$$

What about $f(10,0)$?

$$f(10,0) = \sqrt{75 - 10^2 - 0^2} = \sqrt{75 - 100 - 0}$$

$= \sqrt{-25}$ is not a real number.

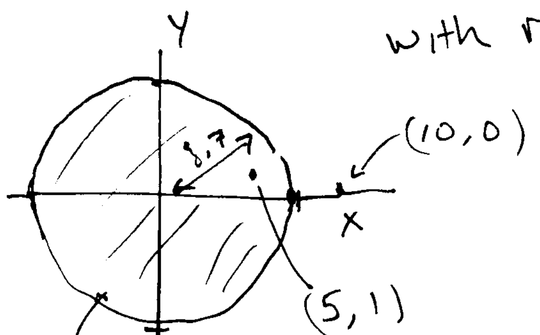
So $(x,y) = (10,0)$ is not in the domain of f.

What is the domain of $f(x,y) = \sqrt{75 - x^2 - y^2}$?

Note: (x,y) must satisfy $75 - x^2 - y^2 \geq 0$

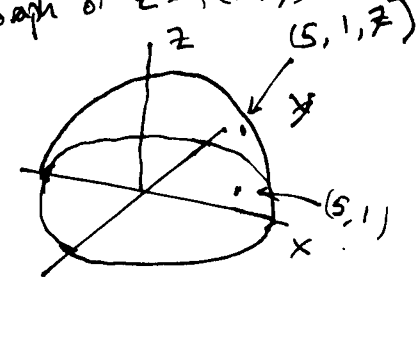
Equivalent: $75 \geq x^2 + y^2$

The Graph of $x^2 + y^2 = 75$ has form $x^2 + y^2 = r^2$ which is a circle centered at $(0,0)$ with radius $r = \sqrt{75} \simeq 8.7$



Domain $= \{ (x,y) \mid x^2 + y^2 \leq 75 \}$

Graph of $z = f(x,y)$



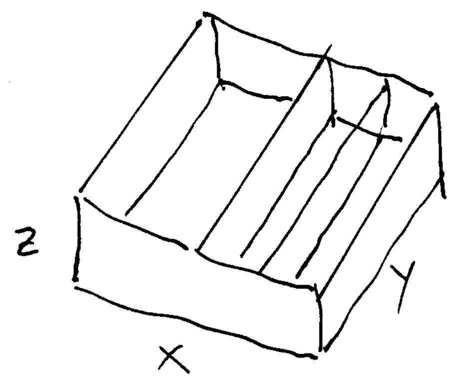
7.1

22) $f(x, y, z) = z\sqrt{x}\ln y$

Find $f(4, e, -1)$.

$f(4, e, -1) = (-1)(\sqrt{4})(\ln e) = (-1)(2)(1) = -2$

36) [Modeling with functions of several variables]



a) Let $V(x,y,z)$ = volume

$V(x,y,z) = xyz$

b) Find a formula for the total area material.

$f(x,y,z)$ = total area (in^2).

4 parallel rectangle of dimension, each of dimension y inches by z inches

2 (front and back) rectangles, each of dimension x in. by z in.

1 bottom of dimension x inches by y inches.

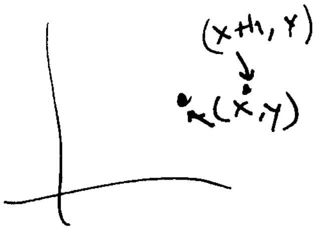
total area $= f(x,y,z) = 4yz + 2xz + xy$

7.2 Partial Derivatives

Defn: $$\frac{\partial}{\partial x} f(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$\frac{\partial}{\partial y} f(x,y) = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

Idea: $f(x,y)$ = temperate at (x,y) on a hot plate.



$f(x+h,y)$ = temp at the nearby point $(x+h, y)$

$f(x+h,y) - f(x,y)$ = difference of temp. (in degrees)

$\dfrac{f(x+h,y) - f(x,y)}{h}$ = average rate of change of temp (in degrees per cm)

$\displaystyle\lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$ = instantaneous rate of change (degrees/cm)

ex: $f(x,y) = x^4 y^2$

acts like a constant (temporarily)

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left[x^4 y^2\right] = y^2 \frac{\partial}{\partial x}\left[x^4\right] = y^2 (4x^3) = 4x^3 y^2$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[x^4 y^2\right] = x^4 \frac{\partial}{\partial y}\left[y^2\right] = x^4 (2y) = 2x^4 y$$

Notation: For functions of <u>one</u> variable, $f(x)$

$$\frac{df}{dx} = f'(x)$$

For two or more variables, $f(x,y)$

$$\frac{\partial f}{\partial x} = f_x(x,y) \quad \text{and} \quad \frac{\partial f}{\partial y} = f_y(x,y)$$

14) $f(x,y) = \ln(xy^3)$ [Hard way first.]

by the chain Rule:

$$\frac{\partial f}{\partial x} = f_x(x,y) = \frac{\partial}{\partial x}\left[\ln(xy^3)\right]$$

$$= \frac{1}{xy^3} \cdot \frac{\partial}{\partial x}\left[xy^3\right] = \frac{1}{\cancel{x}\cancel{y^3}} \cdot \cancel{y^3} = \frac{1}{x}$$

$$\frac{\partial f}{\partial y} = f_y(x,y) = \frac{\partial}{\partial y}\left[\ln(xy^3)\right]$$

$$= \frac{1}{xy^3} \cdot \frac{\partial}{\partial y}\left[xy^3\right] = \frac{1}{xy^3} \cdot 3xy^2 = \frac{3}{y}$$

[Easier way.] Use $f(x,y) = \ln(xy^3)$

$$= \ln x + \ln y^3 = \ln x + 3\ln y$$

$$\frac{\partial f}{\partial x} = f_x(x,y) = \frac{\partial}{\partial x}\left[\ln x + 3\ln y\right] = \frac{1}{x} + 0 = \frac{1}{x}$$

$$\frac{\partial f}{\partial y} = f_y(x,y) = \frac{\partial}{\partial y}\left[\ln x + 3\ln y\right] = 0 + 3\cdot\frac{1}{y} = \frac{3}{y}$$

ex: $f(x,y) = y e^{3xy}$

$f_x(x,y) = \frac{\partial}{\partial x}[y e^{3xy}] = y \frac{\partial}{\partial x}[e^{3xy}]$

$= y e^{3xy} \underbrace{\frac{\partial}{\partial x}[3xy]}_{3y}$

$= \boxed{3y^2 e^{3xy}}$

$f_y(x,y) = \frac{\partial}{\partial y}[y e^{3xy}]$ so by PRODUCT RULE

$= \left(\frac{\partial y}{\partial y}\right) \cdot e^{3xy} + y \cdot \frac{\partial}{\partial y}[e^{3xy}]$

$= 1 \cdot e^{3xy} + y \cdot e^{3xy} \cdot \underbrace{\frac{\partial}{\partial y}[3xy]}_{3x}$

$= \boxed{e^{3xy} + 3xy e^{3xy}}$

28) $f(x,y) = 4x^2 - 3x^3y^2 + 5y^2$

$f_x(x,y) = 8x - 9x^2y^2$ → $f_{xx}(x,y) = 8 - 18xy^2$

→ $f_{xy}(x,y) = -18x^2y$

$f_y(x,y) = -6x^3y + 10y$ → $f_{yx}(x,y) = -18x^2y$ equal

→ $f_{yy}(x,y) = -6x^3 + 10$