7.2 Partial Derivatives (loose ends)

40) Find $f_x$, $f_y$, $f_z$:

$$f(x,y,z) = \ln(x^2 - y^3 + z^4)$$

$$\frac{\partial f}{\partial x} = f_x(x,y,z) = \frac{\partial}{\partial x}\left[\ln(x^2-y^3+z^4)\right]$$

$$= \frac{1}{x^2-y^3+z^4} \cdot \frac{\partial}{\partial x}(x^2-y^3+z^4)$$

$$= \frac{1}{x^2-y^3+z^4} \cdot (2x - 0 + 0) = \frac{2x}{x^2-y^3+z^4}$$

$$\frac{\partial f}{\partial y} = f_y(x,y,z) = \frac{\partial}{\partial y}\left[\ln(x^2-y^3+z^4)\right]$$

$$= \frac{1}{x^2-y^3+z^4} \cdot \underbrace{\frac{\partial}{\partial y}(x^2-y^3+z^4)}_{(-3y^2)} = \frac{-3y^2}{x^2-y^3+z^4}$$

$$\frac{\partial f}{\partial z} = f_z(x,y,z) = \frac{1}{x^2-y^3+z^4} \cdot \underbrace{\frac{\partial}{\partial z}(x^2-y^3+z^4)}_{4z^3} = \frac{4z^3}{x^2-y^3+z^4}$$

24) $g(x,y) = (xy-1)^5$

a) Find $g_x(1,0)$.

$$g_x(x,y) = 5(xy-1)^4 \cdot \underbrace{\frac{\partial}{\partial x}(xy-1)}_{y}$$

$$= 5(xy-1)^4 \cdot y$$

$$g_x(1,0) = 5(1\cdot 0 - 1)^4 \cdot 0 = 0$$

b) $g_y(x,y) = 5(xy-1)^4 x$

$$g_y(1,0) = 5(1\cdot 0 - 1)^4 \cdot 1 = 5$$

46) Profit (dollars) $= P(x, y) = 3x^2 - 4xy + 4y^2 + 80x + 100y + 200$

where $x$ = number of DVD players

$y$ = " " CD "

a) Find the marginal profit function for DVD players.

$$P_x(x, y) = 6x - 4y + 80$$

b) Evaluate and interpret $P_x(200, 100) = 6(200) - 4(100) + 80$

$= 1200 - 400 + 80 = 880$ dollars/DVD player

Interpretation: Amount of added profit from changing from 200 DVDs and 100 CDs to 201 DVDs and 100 CDs.

7.3 Optimizing functions of Several Variables (usually two)

Defn: A point $(a, b)$ is a critical point of $f(x,y)$ if

$$f_x(a, b) = 0 \quad \text{and} \quad f_y(a, b) = 0$$

Remark: Relative max and min values can only occur at critical points.

That is, a list of all critical points will provide candidates of locations of relative max or mins.

Find the critical points:

ex 6) $f(x,y) = 2xy - 2x^2 - 3y^2 + 4x - 12y + 5$

$$\begin{cases} 0 = f_x(x,y) = 2y - 4x + 4 \\ 0 = f_y(x,y) = 2x - 6y - 12 \end{cases}$$

equivalent system of linear equations

$$\begin{cases} -4x + 2y = -4 \\ 2x - 6y = 12 \end{cases}$$

1st: $\quad -4x + 2y = -4$

$2\cdot(2^{nd})$: $\quad 4x - 12y = 24$

$$-10y = 20 \Rightarrow y = -2$$

use $2^{nd}$: $\quad 2x - 6(-2) = 12$

$$2x + 12 = 12$$
$$2x = 0 \Rightarrow x = 0$$

only one critical point: $(x,y) = (0,-2)$

16) $f(x,y) = x^3 - y^2 - 3x + 6y$

$0 = f_x = 3x^2 - 3 = 3(x-1)(x+1) \Rightarrow x = 1$ or $-1$

$0 = f_y = -2y + 6 = -2(y-3) \Rightarrow y = 3$

Two critical points: $(1,3)$ and $(-1,3)$

[Remark: Quiz next time on §7.1 and §7.2]