7.4 Least Squares...

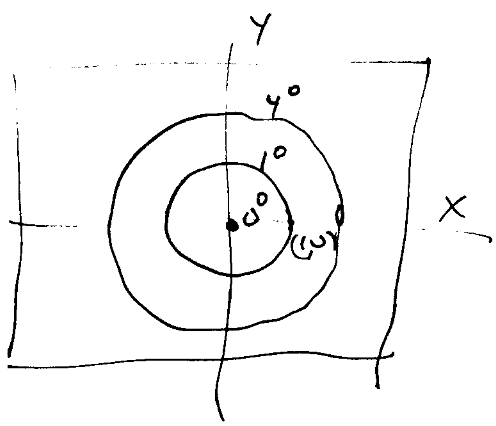
is an interesting topic, but we will skip it.

You might read it on your own.

7.5 Lagrange Multipliers an Constrained optimization

ex: [Thought experiment] An ant on a hot plate wants to minimize its temperature.

Temperature function $= f(x,y) = x^2 + y^2$



where is $f(x,y) = 1$?

$$x^2 + y^2 = 1$$

where is $f(x,y) = 4$?

$$x^2 + y^2 = 4$$

where ~~is~~ are the critical points?

$$0 = f_x(x,y) = 2x$$
$$0 = f_y(x,y) = 2y$$

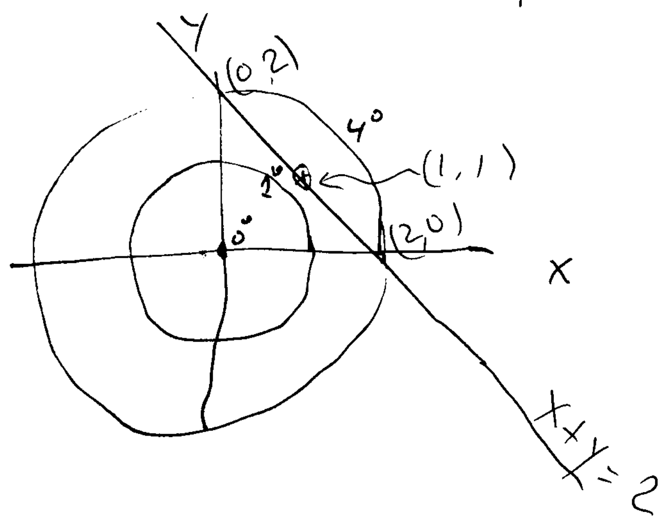
$(x,y) = (0,0)$ is the only critical point. and $f(0,0)$ is a relative (and absolute) minimum, by the Δ-Test.

Remark. This is "unconstrained optimization."

Now, we will consider constrained optimization.

With the same function $f(x,y) = x^2 + y^2$

the ant is ant is only allowed to wander along the line $x + y = 2$.



ex: Solve this problem by the method of Lagrange Multiples.

Given: $f(x,y) = x^2 + y^2$ is objective function
(the function to be minimized)

Constraint: $x + y = 2$ or equivalently
$x + y - 2 = 0$
so let $g(x,y) = x + y - 2$.

Now let $F(x, y, \lambda) = f(x,y) + \lambda g(x,y)$

that is, $F(x, y, \lambda) = x^2 + y^2 + \lambda(x + y - 2)$

and find the critical point(s) of F.

$$F(x,y,\lambda) = x^2+y^2+\lambda(x+y-2)$$

$$\begin{cases} ① \quad 0 = F_x(x,y,\lambda) = 2x + \lambda \\ ② \quad 0 = F_y(x,y,\lambda) = 2y + \lambda \\ ③ \quad 0 = F_\lambda(x,y,\lambda) = x+y-2 \end{cases}$$

← the constraint equation

Solve ① and ② for λ:

$$\left.\begin{aligned} \lambda &= -2x \\ \lambda &= -2y \end{aligned}\right\} \Rightarrow \quad -2x = -2y$$

so $x = y$

plug into ③:

$$x + x - 2 = 0$$
$$2x - 2 = 0$$
$$2x = 2$$
$$\boxed{x = 1 \quad \text{so} \quad y = 1}$$

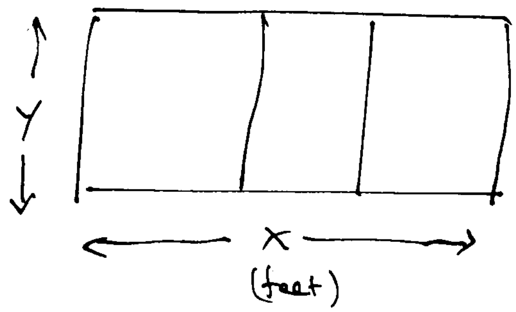
(we don't need this but $\lambda = -2 \cdot 1 = -2$)

The ant is coolest when it location is at

$$(x,y) = (1,1)$$

where $f(1,1) = 1^2 + 1^2 = 2$

§7.5 #26)



maximize the lot enclosed by 12,000 feet of fencing.

objective function: area $= f(x,y) = xy$

constraint: Total length $= 2x + 4y = 12{,}000$

so $g(x,y) = 2x + 4y - 12{,}000 = 0$

$$F(x,y,\lambda) = f(x,y) + \lambda g(x,y)$$
$$= xy + \lambda(2x + 4y - 12{,}000)$$

1 $0 = F_x(x,y,\lambda) = y + 2\lambda$

2 $0 = F_y(x,y,\lambda) = x + 4\lambda$

3 $0 = F_\lambda(x,y,\lambda) = 2x + 4y - 12{,}000$

From 1 $\lambda = -\frac{y}{2}$

From 2 $\lambda = -\frac{x}{4}$

$\Rightarrow -\frac{y}{2} = -\frac{x}{4}$ so

$2y = x$ Now sub into 3

$2(2y) + 4y = 12{,}000$

$8y = 12{,}000 \Rightarrow y = \frac{12{,}000}{8} = \boxed{1{,}500 \text{ feet}}$

$x = 2y = 2(1{,}500) = \boxed{3{,}000 \text{ feet}}$

Mize
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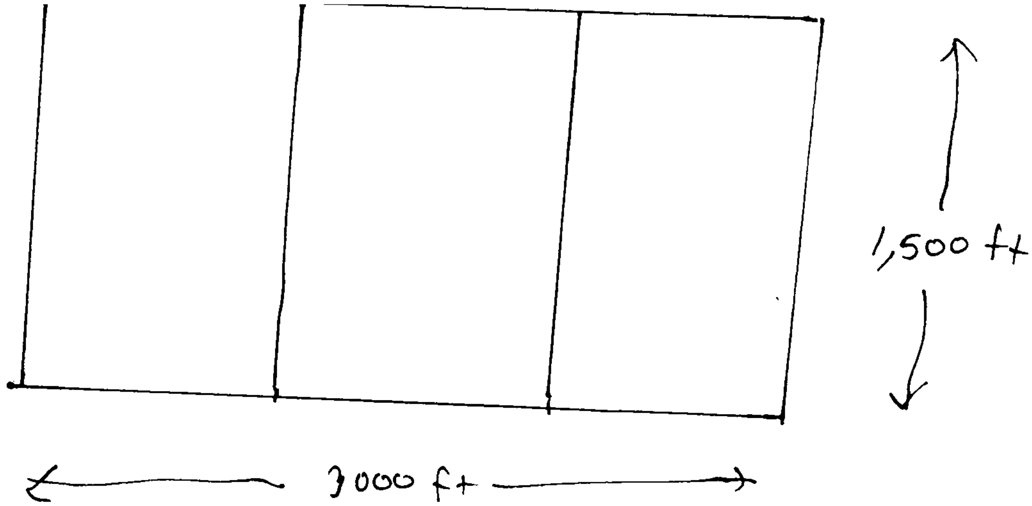

→ total "horizontal" fencing = 3,000 + 3,000 = 6,000 ft
→ total "vertical" fencing = 1500 + 1500 + 1500 + 1500 = 6,000 ft