7.6 Total differentials and Approximation (cont'd)

<u>Warmup</u>: Find the total differentials:

3) $f(x,y) = 6x^{1/2}y^{1/3} + 8$

$$df = f_x \cdot dx + f_y \cdot dy$$
$$= 6 \cdot \tfrac{1}{2} x^{-1/2} y^{1/3}\, dx + 6 \cdot \tfrac{1}{3} x^{1/2} y^{-2/3}\, dy$$
$$= 3x^{-1/2}y^{1/3}\, dx + 2x^{1/2}y^{-2/3}\, dy$$

6) $g(x,y) = \dfrac{x}{x+y}$

$$dg = g_x \cdot dx + g_y \cdot dy$$
$$= \frac{1\cdot(x+y) - x\cdot 1}{(x+y)^2} \cdot dx + \frac{0\cdot(x+y) - x \cdot 1}{(x+y)^2} \cdot dy$$
$$= \boxed{\frac{y}{(x+y)^2}\, dx - \frac{x}{(x+y)^2}\, dy}$$
$$= \frac{y\,dx - x\,dy}{(x+y)^2}$$

18) $f(x,y,z) = \ln(x^2+y^2+z^2)$

$$df = f_x \cdot dx + f_y \cdot dy + f_z \cdot dz$$

$$= \frac{1}{x^2+y^2+z^2} \cdot 2x \cdot dx + \frac{1}{x^2+y^2+z^2} \cdot 2y\,dy + \frac{1}{x^2+y^2+z^2} \cdot 2z\,dz$$

$$= \boxed{\frac{2x\,dx + 2y\,dy + 2z\,dz}{x^2+y^2+z^2}}$$

[end of warmup]

31) [word problem] S = stopping distance (ft)

w = wt of truck (tons)

v = speed (mph)

$$\boxed{S = 0.027\, w\, v^2}$$

what is dS if $(w,v) =$ (4 tons, 60 mph)

and $(\Delta w, \Delta v) = (dw, dv) = (\frac{1}{2}\text{ ton}, 5\text{ mph})$

$$dS = \frac{\partial S}{\partial w}\, dw + \frac{\partial S}{\partial v}\, dv$$

$$= 0.027\, v^2\, dw + 0.054\, w v\, dv$$

For our values:

$$dS = 0.027(60)^2(.5) + 0.054(4)(60)(5)$$

$$= 0.027(1800) + 0.054(1200)$$

$$= 113.4 \text{ feet}$$

7.7 Multiple Integrals

<u>Idea</u>: Iterated integrals

<u>ex</u>: [sort of a "partial antiderivative"]

$$\int_0^2 6xy^2\,dx = y^2\int_{x=0}^{x=2} 6x\,dx$$

$$= y^2\cdot 3x^2\Big|_{x=0}^{x=2} = y^2\cdot 3\cdot 2^2 - y^2\cdot 3\cdot 0^0$$

$$= 12y^2$$

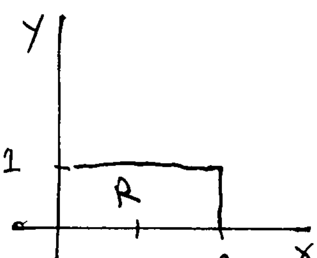
<u>ex</u>: $$\int_0^1 6xy^2\,dy = 6x\int_{y=0}^{y=1} y^2\,dy$$

$$= 6x\cdot\frac{y^3}{3}\Big|_{y=0}^{y=1} = 2xy^3\Big|_{y=0}^{y=1}$$

$$= 2x\cdot 1^3 - 2x\cdot 0^3 = 2x$$

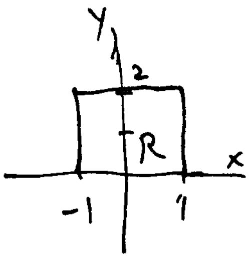
<u>ex</u>: $$\int_0^1\int_0^2 6xy^2\,dx\,dy = \int_0^1\left[\int_0^2 6xy^2\,dx\right]dy$$

$$= \int_0^1 12y^2\,dy$$



$$= 4y^3\Big|_0^1 = 4\cdot 1^3 - 4\cdot 0^3 = \boxed{4}$$

16) $$\int_{-1}^{1}\int_{0}^{2} (2x^2+y^2)\,dy\,dx$$



$$= \int_{-1}^{1} \left(2x^2y + \frac{1}{3}y^3\right)\Big|_{y=0}^{2} \, dx$$

$$= \int_{-1}^{1} \left[\left(2x^2\cdot 2 + \frac{1}{3}\cdot 2^3\right) - 0\right] dx$$

$$= \int_{-1}^{1} \left(4x^2 + \frac{8}{3}\right) dx$$

$$= \left(\frac{4}{3}x^3 + \frac{8}{3}x\right)\Big|_{-1}^{1}$$

$$= \left(\frac{4}{3}\cdot 1^3 + \frac{8}{3}\cdot 1\right) - \left(\frac{4}{3}(-1)^3 + \frac{8}{3}\cdot(-1)\right)$$

$$= \left(\frac{4}{3} + \frac{8}{3}\right) - \left(-\frac{4}{3} - \frac{8}{4}\right)$$

$$= 4 + 4 = \boxed{8}$$