7.7 Double Integrals (cont'd)

2) $$\int_{x=1}^{x=y^2} 10x^4\,dx$$ ← so x is the variable of integration

$$= 2x^5 \Big|_{x=1}^{x=y^2} = 2(y^2)^5 - 2\cdot 1^5$$

$$= 2y^{10} - 2$$ ← Note: All x's disappeared.

6) $$\int_0^y (4x-y)\,dx = (2x^2 - xy)\Big|_{x=0}^{x=y}$$

$$= (2\cdot y^2 - y\cdot y) - (2\cdot 0^2 - 0\cdot y)$$

$$= 2y^2 - y^2 - 0 = y^2$$

24) $$\int_2^4 \int_0^x (x-2y)\,dy\,dx$$

$$= \int_2^4 (xy - y^2)\Big|_{y=0}^{y=x}\,dx$$

$$= \int_2^4 \left[(x^2 - x^2) - (0 - 0^2)\right] dx = \int_2^4 0\,dx$$

$$= 0$$

28) $$\int_0^2 \int_{-x}^{x} (x^2 - y)\, dy\, dx$$

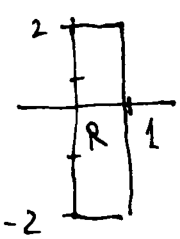
$$= \int_0^2 \left(x^2 y - \frac{1}{2} y^2\right)\Big|_{y=-x}^{x} dx$$

$$= \int_0^2 \left[\left(x^2\cdot x - \frac{1}{2}x^2\right) - \left(x^2\cdot(-x) - \frac{1}{2}(-x)^2\right)\right] dx \quad \leftarrow \text{No y's.}$$

$$= \int_0^2 \left[x^3 - \frac{1}{2}x^2 + x^3 + \frac{1}{2}x^2 \right] dx \qquad \text{No x or y.}$$

$$= \int_0^2 2x^3\, dx = \frac{1}{2}x^4 \Big|_0^2 = \frac{1}{2}\cdot 2^4 - \frac{1}{2}\cdot 0^4 = 8$$

32) Take $R = \{(x,y) \mid 0 \le x \le 1,\ -2 \le y \le 2\}$



and calculate $\iint_R x e^y\, dx\, dy$. We can calculate

Either (i) $$\int_{-2}^{2} \int_0^1 x e^y\, dx\, dy$$

OR (2) $$\int_0^1 \int_{-2}^{2} x e^y\, dy\, dx.$$

32 cont'd) Let's calculate 1:

$$\int_{-2}^{2} \int_{0}^{1} x e^{y}\, dx\, dy$$

$$= \int_{-2}^{2} e^{y} \cdot \int_{0}^{1} x\, dx\, dy$$

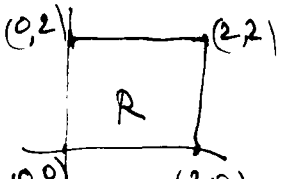
$$= \int_{-2}^{2} e^{y} \cdot \left(\frac{x^2}{2}\right)\Big|_{x=0}^{1} dy$$

$$= \int_{-2}^{2} e^{y} \cdot \left[\frac{1}{2} - 0\right] dy = \int_{-2}^{2} \frac{1}{2} e^{y}\, dy$$

$$= \frac{1}{2} e^{y} \Big|_{-2}^{2} = \boxed{\frac{1}{2} e^{2} - \frac{1}{2} e^{-2}}$$

<u>Remark</u>: Had we calculated ~~the~~ the iterated integral 2 we'd have gotten the same answer.

36) $f(x,y) = x^2 + y^2$. Find the <u>volume</u> under the graph of $f(x,y)$ which lies over $R = \{(x,y) \mid 0 \le x \le 2,\ 0 \le y \le 2\}$



$$\text{vol.} = \iint_R (x^2+y^2)\, dx\, dy = \int_0^2 \int_0^2 (x^2+y^2)\, dy\, dx$$

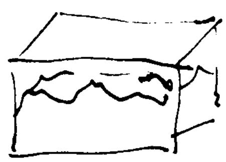
$$= \int_0^2 \left(x^2 y + \frac{1}{3} y^3\right)\Big|_{y=0}^{2} dx = \int_0^2 \left(2x^2 + \frac{8}{3} - 0\right) dx$$

$$= \left(\frac{2}{3} x^3 + \frac{8}{3} x\right)\Big|_0^2 = \frac{2}{3} \cdot 2^3 + \frac{8}{3} \cdot 2 - 0 = \boxed{\frac{32}{3}}$$

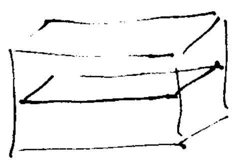
Defn

$$\left\{\begin{matrix}\text{Average} \\ \text{value}\end{matrix}\right\} = \frac{1}{\text{area } R} \iint_R f(x,y)\,dx\,dy$$

Remark:



ice scultpre in an aquarium.
Let ice melt.

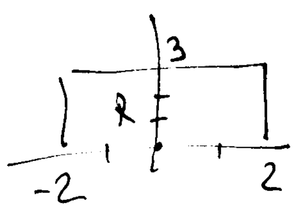


$$\begin{matrix}\text{volume of} \\ \text{ice}\end{matrix} = \begin{matrix}\text{volume of} \\ \text{water}\end{matrix}$$

$$\begin{matrix}\text{depth of} \\ \text{water}\end{matrix} = \frac{(lwh)}{(lw)} = \frac{\text{volume of water}}{\text{area of base}}$$

$$= \frac{\iint_R f(x,y)\,dx\,dy}{\text{area of } R}$$

41) avg temp $= f(x,y) = 48 + 4x - 2y$



$$\begin{matrix}\text{avg} \\ \text{temp} \\ \text{over } R\end{matrix} = \frac{1}{\text{area of } R} \iint_R (48 + 4x - 2y)\,dx\,dy$$

$$= \frac{1}{12 \text{ miles}^2} \int_{-2}^{2}\int_{0}^{3} (48 + 4x - 2y)\,dy\,dx$$