7.7 (Loose ends) in multiple integrals

§7.7 32) again

$$\int_{-2}^{2}\int_{0}^{1} x e^{y}\, dx\, dy$$

because e^y acts like a constant in the inner integral:

$$= \int_{-2}^{2} e^{y} \int_{0}^{1} x\, dx\, dy = \int_{-2}^{2} e^{y} \underbrace{\left[\int_{0}^{1} x\, dx\right]}_{\text{a genuine constant}} dy$$

$$= \left[\int_{0}^{1} x\, dx\right]\left[\int_{-2}^{2} e^{y}\, dy\right]$$

$$= \left(\frac{x^2}{2}\right)\Big|_{0}^{1} \quad \left(e^{y}\right)\Big|_{-2}^{2}$$

$$= \left(\frac{1}{2} - 0\right)\cdot\left(e^{2} - e^{-2}\right) = \frac{1}{2}\left(e^{2} - e^{-2}\right)$$

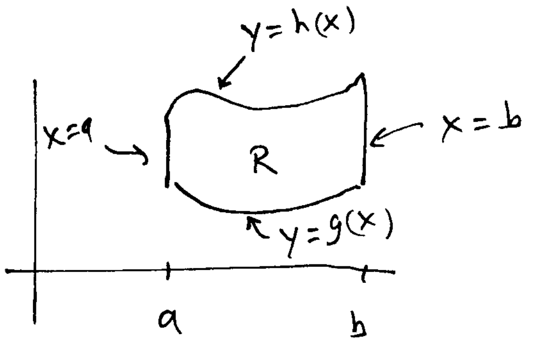
MORAL: If (1) the limits in the double integral are genuine constants (equivalent: R is a rectangle)

and (2) the integrand $f(x,y)$ can be factored so that $f(x,y) = g(x)\cdot h(y)$

(e.g. $f(x,y) = xe^{y} = (x)(e^{y})$ where $g(x) = x$ and $h(y) = e^{y}$)

THEN $\displaystyle\iint_R f(x,y)\, dx\, dy = \int_a^b g(x)\, dx \cdot \int_c^d h(y)\, dy$.

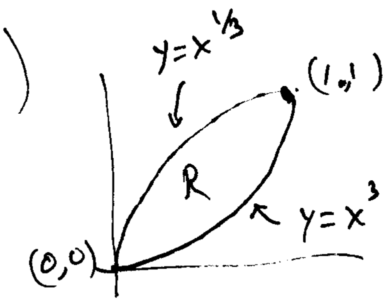
If R is not a rectangle: The iterated integral can be set up this way:



$$\iint_R f(x,y)\,dx\,dy$$

$$= \int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dy\,dx$$

38)



$$\iint_R 3xy^2\,dx\,dy$$

$$= \int_0^1 \int_{x^3}^{x^{1/3}} 3xy^2\,dy\,dx$$

$$= \int_0^1 x \int_{x^3}^{x^{1/3}} 3y^2\,dy\,dx = \int_0^1 x\cdot(y^3)\Big|_{x^3}^{x^{1/3}}\,dx$$

$$= \int_0^1 x\left[(x^{1/3})^3 - (x^3)^3\right]dx$$

$$= \int_0^1 x\left[x - x^9\right]dx = \int_0^1 (x^2 - x^{10})\,dx$$

$$= \left(\frac{1}{3}x^3 - \frac{1}{11}x^{11}\right)\Big|_0^1 = \left(\frac{1}{3} - \frac{1}{11}\right) - 0 = \frac{11-3}{33}$$

$$= \frac{8}{33}$$