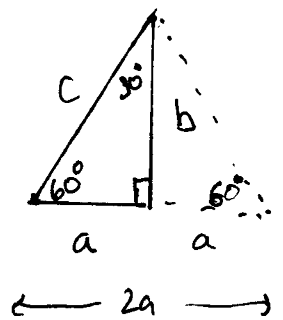
8.1 Triangles, Radian Measure

$30^\circ - 60^\circ - 90^\circ$



$$\boxed{c = 2a}$$

$$a^2 + b^2 = c^2$$

$$a^2 + b^2 = (2a)^2$$

$$a^2 + b^2 = 4a^2$$

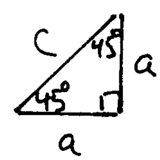
$$b^2 = 3a^2$$

$$\sqrt{b^2} = \sqrt{3a^2}$$

$$\boxed{b = a\sqrt{3}}$$

Remark:
If you know one of a, b or c, you can calculate the other two.

$45^\circ - 45^\circ - 90^\circ$



$$\boxed{a = b}$$

$$a^2 + a^2 = c^2$$

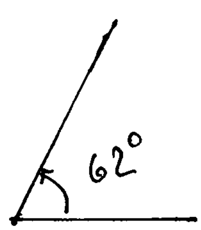
$$2a^2 = c^2$$

$$\sqrt{2a^2} = \sqrt{c^2}$$

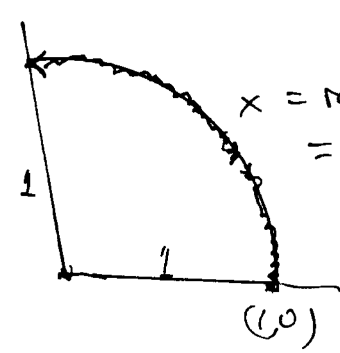
$$\boxed{a\sqrt{2} = c}$$

Remark:
Ditto.

Degree measure



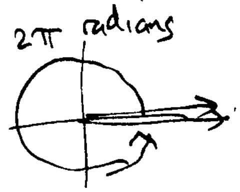
Radian measure



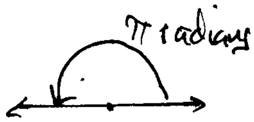
x = radian measure
= "length of string" wrapped around a spool of radius 1.

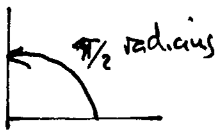
Remark: Since 360 degrees = once around a circle of radius 1

= 2π radians



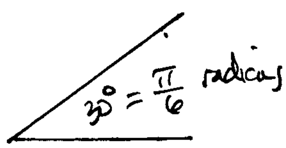
$$\boxed{180 \text{ degrees} = \pi \text{ radians}}$$



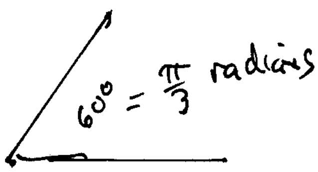


90 degrees = $\frac{\pi}{2}$ radians

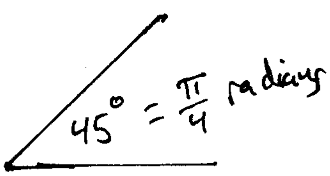
30 degrees = $\frac{\pi}{6}$ radians



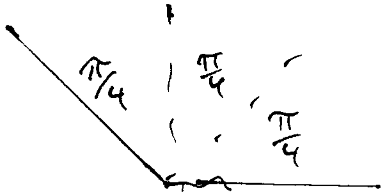
60 degrees = $\frac{\pi}{3}$ radians



45 degrees = $\frac{\pi}{4}$ radians



example: what is 135 degrees in radians? Answer: count



$135^\circ = 3 \cdot 45^\circ$

$= 3 \cdot \frac{\pi}{4} = \frac{3\pi}{4}$ radians

<u>ex</u>: 72 degrees = how many radians?

<u>Use</u>: $\frac{180 \text{ degrees}}{\pi \text{ radians}} = 1$ OR $\frac{\pi \text{ radians}}{180 \text{ degrees}} = 1$

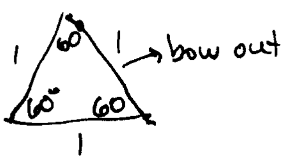
$$72 \cancel{\text{degrees}} \cdot \frac{\pi \text{ radians}}{180 \cancel{\text{degrees}}} = \frac{72\pi}{180} \text{ radians}$$

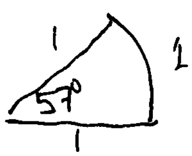
$$= \frac{36\pi}{90} = \frac{12\pi}{30} = \frac{4\pi}{10} = \frac{2\pi}{5} \text{ radians}$$

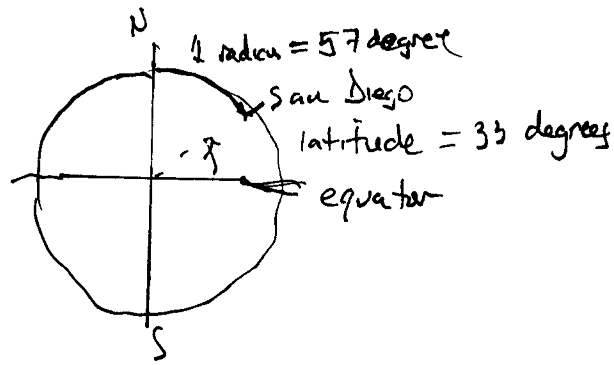
<u>ex</u>: 1 radian = how many degrees?

$$1 \cancel{\text{radian}} \cdot \frac{180 \text{ degrees}}{\pi \cancel{\text{radians}}} = \frac{180}{\pi} \text{ degrees}$$

$$\approx 57.296 \text{ degrees}$$

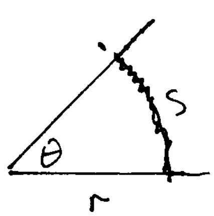






<u>Why Radians?</u>

Because: The arc length cut ~~off~~ by any central angle is



$$\begin{pmatrix}\text{Arc} \\ \text{length}\end{pmatrix} = (\text{Radians}) \cdot (\text{Radius})$$

$$\boxed{s = \theta r}$$

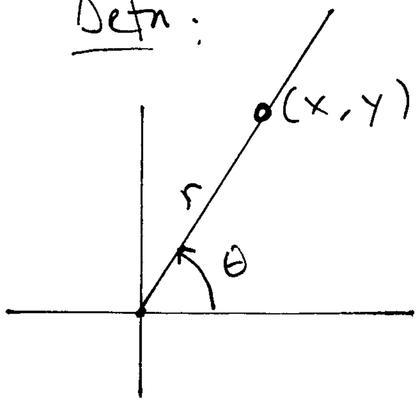
s = arc length

θ = radian measure

r = radius of the arc

8.2 Sine and Cosine Functions

Defn:



$$\sin\theta = \frac{y}{r}$$

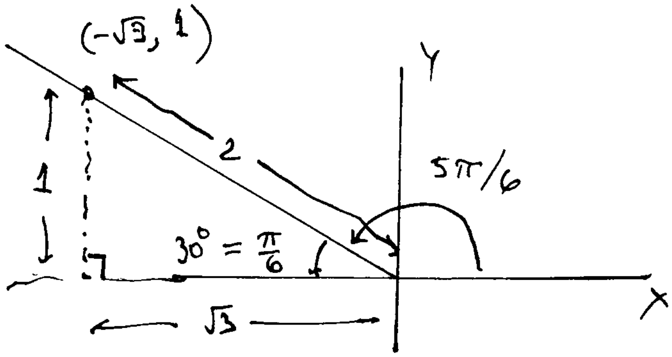
$$\cos\theta = \frac{x}{r}$$

where (x,y) is a point on the terimal ray of the angle with measure θ and r is the distance from (x,y) to the origin.

Remark: It doesn't matter which point you pick on the terminal ray.

ex: [Find sine and cosine of a "famous angle" i.e a multiple of $30^\circ = \pi/6$ or $45^\circ = \pi/4$]

Find $\sin\frac{5\pi}{6}$ and $\cos\frac{5\pi}{6}$ exactly.



$x = -\sqrt{3}$

$y = 1$

$r = 2$ so

$$\sin\frac{5\pi}{6} = \frac{y}{r} = \frac{1}{2}$$

$$\cos\frac{5\pi}{6} = \frac{x}{r} = \frac{-\sqrt{3}}{2}$$

	θ	$\sin\theta$	$\cos\theta$
$0^\circ =$	0	$\frac{\sqrt{0}}{2} = 0$	1
$30^\circ =$	$\frac{\pi}{6}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$45^\circ =$	$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$60^\circ =$	$\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$90^\circ =$	$\frac{\pi}{2}$	$\frac{\sqrt{4}}{2} = 1$	0