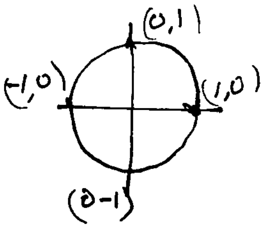
8.2 Sine and cosine (cont'd)

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

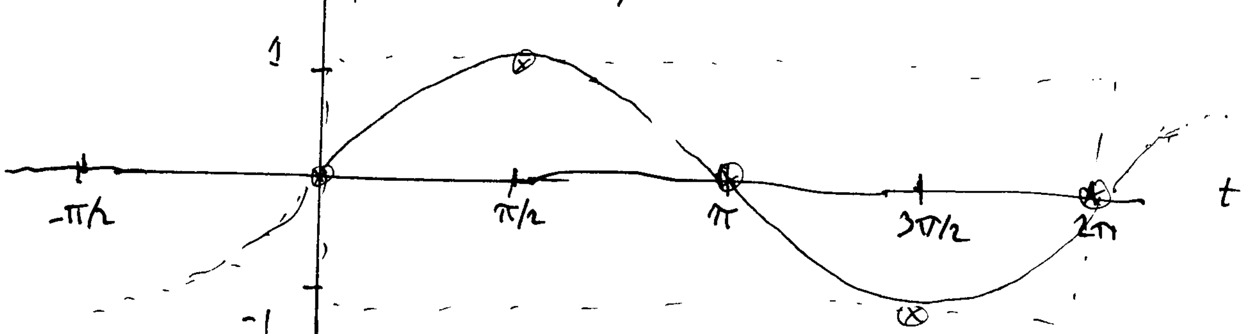
Quadrantal angles



θ	$\sin\theta$	$\cos\theta$
$0 = 0^\circ$	0	1
$\pi/2 = 90^\circ$	1	0
$\pi = 180^\circ$	0	-1
$3\pi/2 = 270^\circ$	-1	0
$2\pi = 360^\circ$	0	1

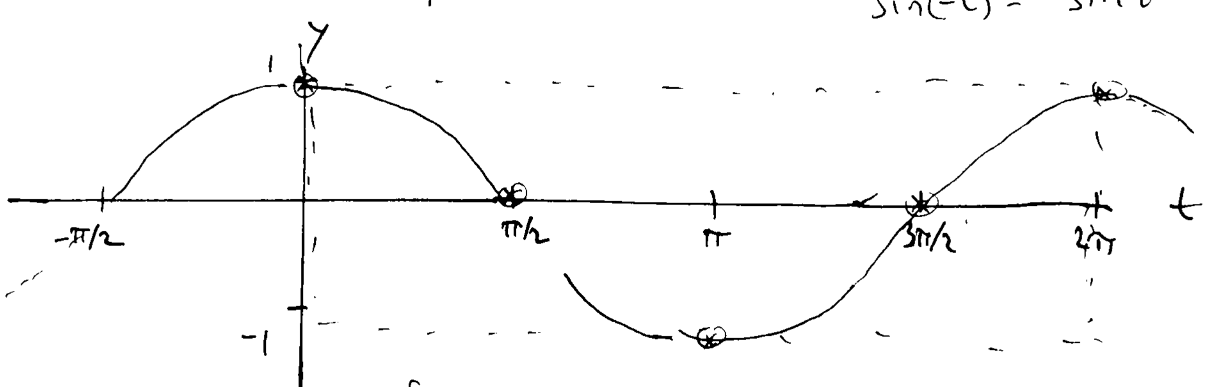
t = independent variable
y = dependent variable

$y = \sin t$



amplitude = 1
period = 2π

sine is an odd function
$\sin(-t) = -\sin t$



cosine is a even function
$\cos(-t) = \cos t$

$y = \cos t$

ex Modified sine function

$$y = 3\sin 2t \qquad \text{amplitude} = 3$$

$$\text{period} = \frac{2\pi}{2} = \pi$$

Reason: Standard sine function one cycle corresponds to $0 \le t \le 2\pi$

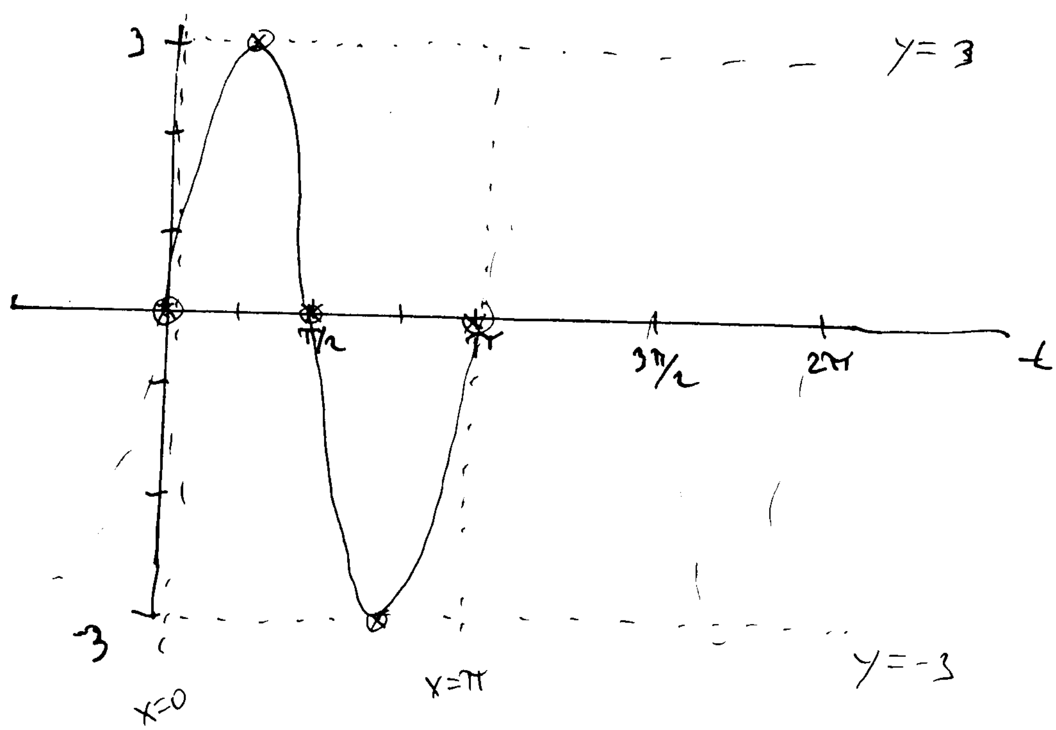
But for $y = 3\sin 2t$, one cycle corresponds to

$$0 \le 2t \le 2\pi$$

$$\frac{0}{2} \le \frac{2t}{2} \le \frac{2\pi}{2}$$

$$0 \le t \le \pi$$

$y = 3\sin 2t$



Modified sine and cosine

For $\begin{cases} y = a \sin bt \\ y = a \cos bt \end{cases}$ or

a = amplitude

$\frac{2\pi}{b}$ = period

Trig identities

$\sin^2 t + \cos^2 t = 1$ Pythagorean identity

ex: $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{2}$

$= \left[\sin\left(\frac{\pi}{6}\right)\right]^2 + \left[\cos\left(\frac{\pi}{6}\right)\right]^2$

$= \left[\frac{1}{2}\right]^2 + \left[\frac{\sqrt{3}}{2}\right]^2$

$= \frac{1}{4} + \frac{3}{4} = 1$

$\left.\begin{array}{l} \sin(t + 2\pi) = \sin t \\ \cos(t + 2\pi) = \cos t \end{array}\right\}$ periodicity

$\left.\begin{array}{l} \sin t = \cos\left(\frac{\pi}{2} - t\right) \\ \cos t = \sin\left(\frac{\pi}{2} - t\right) \end{array}\right\}$ complementary ang6

ex: $\sin \frac{\pi}{6} = \frac{1}{2} = \cos \frac{\pi}{3}$

$= \sin 30^\circ \qquad = \cos 60^\circ$

$\left.\begin{array}{l} \sin(s \pm t) = (\sin s)\cdot(\cos t) \pm (\cos s)\cdot(\sin t) \\ \cos(s \pm t) = (\cos s)\cdot(\cos t) \mp (\sin s)\cdot(\sin t) \end{array}\right\}$ sum and difference identities

8.3 Derivatives of sine and cosine functions

$$\boxed{\begin{aligned} \frac{d}{dt}\sin t &= \cos t \\ \frac{d}{dt}\cos t &= -\sin t \end{aligned}}$$

ex: [Combine with product, quotient, chain rule, etc.]

2) $f(t) = t^2 \cos t$ Find $f'(t)$. (product rule)

$$f'(t) = \frac{d}{dt}[t^2] \cdot \cos t + t^2 \cdot \frac{d}{dt}[\cos t]$$
$$= 2t\cos t + t^2(-\sin t)$$
$$= 2t\cos t - t^2 \sin t$$

6) $f(t) = \cos(t^3 + t + 1)$ (chain rule)

$$f'(t) = -\sin(t^3+t+1) \cdot \frac{d}{dt}[t^3+t+1]$$
$$= [-\sin(t^3+t+1)](3t^2+1)$$
$$= -(3t^2+1)\sin(t^3+t+1)$$

14) a) $f(t) = t\cos \pi t$

$$f'(t) = \frac{d}{dt}[t] \cdot \cos \pi t + t \cdot \frac{d}{dt}[\cos \pi t]$$
$$= 1 \cdot \cos \pi t + t \cdot [-\sin(\pi t) \cdot \underbrace{\tfrac{d}{dt}(\pi t)}_{\pi}]$$
$$= \cos \pi t - \pi t \sin \pi t$$

b) $f'(0) = \cos 0 - \pi \cdot 0 \cdot \sin 0 = 1$