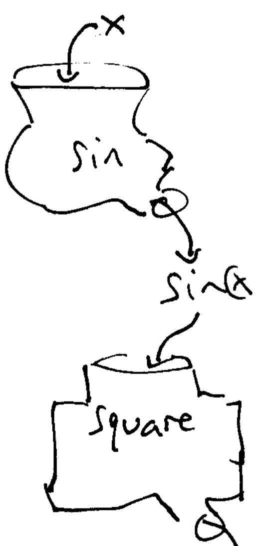
8.3 Derivatives of sine and cosine (cont'd)

Remark (Note on notation) If $y = f(x)$ is a function

then $y = f^2(x)$ is a new function

defined by $f^2(x) = [f(x)]^2$.

ex: $\sin^2 x = [\sin(x)]^2$



$[\sin(x)]^2 = \sin^2(x)$

$$\sin^2 \frac{\pi}{4} = \left[\sin\left(\frac{\pi}{4}\right)\right]^2 = \left[\frac{\sqrt{2}}{2}\right]^2 = \frac{2}{4} = \frac{1}{2}$$

ex: Find $\frac{d}{dx}[\sin^2 x] = \frac{d}{dx}\left[(\sin x)^2\right]$

$$= 2(\sin x)^1 \cdot \frac{d}{dx}[\sin x]$$

$$= 2(\sin x)(\cos x) = 2 \sin x \cos x$$

ex: $f(x) = \cos^3(x^2+1)$ Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[\cos(x^2+1)\right]^3$$
$$= 3\left[\cos(x^2+1)\right]^2 \cdot \frac{d}{dx}\left[\cos(x^2+1)\right]$$
$$= 3\left[\cos(x^2+1)\right]^2 \cdot \left[-\sin(x^2+1)\right] \cdot \frac{d}{dx}(x^2+1)$$
$$= 3\left[\cos(x^2+1)\right]^2 \cdot \left[-\sin(x^2+1)\right] \cdot (2x)$$
$$= -6x\cos^2(x^2+1) \cdot \sin(x^2+1)$$

52) a) $f(t) = \dfrac{1-\cos t}{\sin t}$ Find $f'(t)$.

$$f'(t) = \frac{\sin t \cdot \frac{d}{dt}[1-\cos t] - (1-\cos t) \cdot \frac{d}{dt}[\sin t]}{\sin^2 t}$$

$$= \frac{\sin t \cdot \sin t - (1-\cos t)\cos t}{\sin^2 t}$$

$$= \frac{\sin^2 t - \cos t + \cos^2 t}{\sin^2 t} \overset{\text{optional}}{=} \frac{1-\cos t}{\sin^2 t}$$

[using: $\sin^2 t + \cos^2 t = 1$]

52b) Evaluate $f'\left(\frac{\pi}{2}\right)$

$$f'(t) = \frac{1-\cos t}{\sin^2 t}$$

$$f'\left(\frac{\pi}{2}\right) = \frac{1-\cos\left(\frac{\pi}{2}\right)}{\left[\sin\left(\frac{\pi}{2}\right)\right]^2} = \frac{1-0}{1^2} = 1$$

Do these:

Find $f'(x)$ where

58) $f(x) = x^2\cos x - 2x\sin x - 2\cos x$

and 67) Find $f''(x)$ if $f(x) = e^{\sin x}$, the second derivative.

solutions:

58) $$f'(x) = \cos x \frac{d}{dx}[x^2] + x^2\frac{d}{dx}[\cos x] - \left(\sin x\frac{d}{dx}[2x] + 2x\frac{d}{dx}[\sin x]\right) - \frac{d}{dx}[2\cos x]$$

$$= 2x\cancel{\cos x} - x^2\sin x - 2\cancel{\sin x} - \cancel{2x\cos x} + 2\cancel{\sin x}$$

$$= \boxed{-x^2\sin x}$$

67) $$f(x) = e^{\sin x}$$

$$f'(x) = e^{\sin x}\cdot\frac{d}{dx}[\sin x] = \cos x\, e^{\sin x}$$

$$f''(x) = e^{\sin x}\cdot\frac{d}{dx}[\cos x] + \cos x\cdot\frac{d}{dx}\left[e^{\sin x}\right]$$

$$= -\sin x\cdot e^{\sin x} + \cos x\cdot e^{\sin x}\cdot\underbrace{\frac{d}{dx}[\sin x]}_{\cos x}$$

$$= \boxed{-\sin x\cdot e^{\sin x} + \cos^2 x\cdot e^{\sin x}}$$

8.4 Integrals of sine and cosine

$$\boxed{\begin{aligned}\int \sin t\, dt &= -\cos t + C \\ \int \cos t\, dt &= \sin t + C\end{aligned}}$$

ex: $\int (3x^2 + 2x + 4\sin x + 3\cos x)\, dx$

$= x^3 + x^2 - 4\cos x + 3\sin x + C$

ex (parts) $\int u\,dv = uv - \int v\,du$

$\int x \cos x\, dx = x \sin x - \int \sin x\, dx$

$u = x$	$dv = \cos x\, dx$
$du = dx$	$v = \sin x$

$= x\sin x - (-\cos x) + C$

$= x \sin x + \cos x + C$

Modified Basic Formulas

$$\int \sin(ax+b)\, dx = \frac{-1}{a} \cos(ax+b) + C$$

$$\int \cos(ax+b)\, dx = \frac{1}{a} \sin(ax+b) + C$$

ex:

$$\int x^2 \cos 3x \, dx = (*)$$

signs	diff	int
$+$	x^2	$\cos 3x$
$-$	$2x$	$\frac{1}{3}\sin 3x$
$+$	2	$-\frac{1}{9}\cos 3x$
	0	$-\frac{1}{27}\sin 3x$

$$(*) = (x^2)\left(\tfrac{1}{3}\sin 3x\right) - (2x)\left(\tfrac{-1}{9}\cos 3x\right) + (2)\left(\tfrac{-1}{27}\sin 3x\right) + C$$

$$= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x + C$$