8.4 Integral of sine and cosine (cont'd)

(especially: by substitution)

12) $\int \cos(1-t)\,dt$

Method 1: Modified basic formula (recommended)

$$\int \cos(-t+1)\,dt = \int \cos(at+b)\,dt$$

where $a=-1$, $b=1$.

$$= \frac{1}{-1}\sin(-t+1) + C = -\sin(-t+1) + C$$

Method 2: By substitution (like textbook)

$$\int \cos(1-t)\,dt = \frac{1}{-1}\int \cos(1-t)(-dt)$$

$u = 1-t$
$du = -dt$

$$= -\int \cos u\,du$$

$$= -\sin u + C = -\sin(1-t) + C$$

16) $\int \cos^3 t \sin t\,dt = \int (\cos t)^3 \sin t\,dt$

$u = \cos t$
$du = -\sin t\,dt$

$$= -\int (\cos t)^3 (-\sin t)\,dt$$

$$= -\int u^3\,du = -\frac{1}{4}u^4 + C$$

$$= -\frac{1}{4}\cos^4 t + C$$

18) $\int \sqrt{\sin t}\,\cos t\,dt = \int (\sin t)^{1/2}\cos t\,dt$

$u = \sin t$

$du = \cos t\,dt$

$= \int u^{1/2}\,du$

$= \frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{3}(\sin t)^{3/2} + C}$

$= \frac{2}{3}\sqrt{\sin^3 t} + C$

22) $\int \frac{\cos t}{1+\sin t}\,dt = \int \frac{du}{u} = \ln|u| + C$

$u = 1 + \sin t$

$du = \cos t\,dt$

$= \ln|1+\sin t| + C$

36) $\int_{t=\pi/2}^{t=\pi} e^{\sin t}\cos t\,dt = \int_{u=1}^{u=0} e^u\,du = -\int_0^1 e^u\,du$ ← optional

$u = \sin t$

$du = \cos t\,dt$

$t = \frac{\pi}{2} \Leftrightarrow u = \sin\frac{\pi}{2} = 1$

$t = \pi \Leftrightarrow u = \sin\pi = 0$

$= -e^u\Big|_0^1 = -e^1 - (-e^0)$

$= -e + 1$

$= 1 - e$

≈ -1.7182818284

47) $300 - 300\cos\left(\frac{\pi}{6}t\right)$ = rate that birds fly by (per month)

How many birds pass in the entire year?

$$\int_0^{12} \left(300 - 300\cos\tfrac{\pi}{6}t\right) dt = \int \left(\frac{\text{birds}}{\text{month}}\right)(\text{months}) = \text{birds}$$

$$= \left(300t - 300\cdot\frac{6}{\pi}\sin\frac{\pi}{6}t\right)\Big|_0^{12}$$

$$= \left(300(12) - 300\cdot\frac{6}{\pi}\sin\frac{\pi}{6}\cdot 12\right) - \left(0 - 300\cdot\frac{6}{\pi}\sin\frac{\pi}{6}\cdot 0\right)$$

$$= 3600 - \frac{1800}{\pi}\sin 2\pi + \frac{1800}{\pi}\sin 0 = 3600 \text{ birds}$$

Brief intro to 8.5: Other trig functions

Definitions: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$